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Directional pedestal-free laser Doppler velocimetry without frequency biasing. Part 1

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The spatial structure of the optical field on the detector of a laser Doppler velocimeter is examined. It is shown that for sufficiently small scatterers, the optical field is a traveling wave of shape determined by the detector optics alone. The direction of travel of the optical field reflects that of the scattering particle. Thus, the direction of motion of the particle is determined by temporal correlation of photocurrents from two spatially offset detector arrays. The arrays also eliminate the Doppler pedestal as shown by Ogiwara (1979). In this paper, the theory of the new method is described; experimental implementation will be described in a complementary paper.

I. Introduction

The measurement of localized fluid velocity by the laser Doppler method has developed along classical heterodyne techniques. Yeh and Cummins[1] first obtained the velocity by optically mixing a reference laser beam with Doppler-shifted light scattered by particles carried in the flow. A dual-scatter or dual-differential Doppler method was proposed later, where the scattering particle was illuminated by two beams derived from the same laser and mixing occurred only between the scattered light. This mode, due to superior signal/noise and ease of alignment in most practical situations, is the preferred mode, although in particle-dense flows the coherent or reference mode of Yeh and Cummins is recommended (Ref. 2). The laser Doppler velocimeter (LDV) involves use of a single photosensor as the optical mixer, detecting the full power of the optical signal. The detected photoelectric current varies as a Gaussian modulated sine function on a so-called pedestal,[3] where the Gaussian modulation results from the intensity distribution across the TEM_{00} laser beams.

In either of the two modes described above, there exists a 180° uncertainty in the direction of motion of the scatterer. To resolve this directional ambiguity, rotating radial gratings or Bragg cells must at present be employed. These devices introduce a zero-velocity frequency offset in the linear frequency-to-velocity relation (see, e.g., Ref. 3). A second problem as mentioned earlier is the Doppler pedestal. The Doppler signal must be high-pass filtered and the pedestal removed prior to processing on period counting electronics. In this paper we demonstrate that directional information is contained in the spatial structure of the optical signal at the detector surface, and that, by the use of two offset differential linear arrays, the direction of motion of the scatterer can be determined without the need for frequency biasing devices. The diode arrays, following the principle of Ogiwara,[4] also eliminate the Doppler pedestal giving a zero-mean photocurrent. Finally, the new method requires only one laser beam. Two velocity components can be determined by the use of two orthogonal arrays. The simultaneous elimination of beam splitting optics, directional ambiguity, Doppler pedestal, and optical frequency shifting devices leads to a considerable simplification of the LDV.

The principle of the new method can be simply explained as follows: treating the scattering particle as a small or point light source, fringes are set up on the detector surface through the diffraction of light by the receiving optics. An example of such receiving optics is shown in Fig. 1. We demonstrate below that when these interference fringes are formed from light scattered by a moving particle, they represent a traveling wave (a fringe train) on the detector plane. It is further shown that the direction of travel of the fringes reflects the direction of motion of the scattering particle. Thus the direction of motion of the interference fringes determines the flow direction.

II. Theory

To examine the spatial structure of the optical signal we employ the Fourier formulation valid in the Fraunhofer diffraction approximation (see Ref. 5) in a manner

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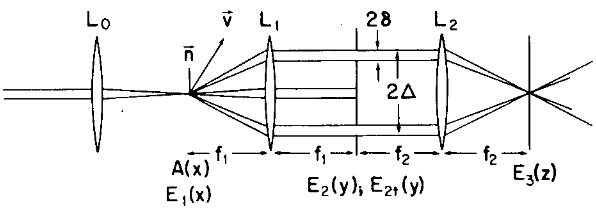

Fig. 1. Schematic of the two-slit directional velocimeter. The intensity distribution in the detector plane, $E_3E_3^*$ is the same as that in Young's experiment, centered at the geometrical image point of the scatterer.

similar to that of Rudd[6] and Ogiwara.[4] For reasons of mathematical simplicity and subsequent physical interpretation in the familiar context of the dual-Doppler LDV, we use the geometry of Fig. 1. A single TEM_{00} laser beam illuminates a particle in plane 1, which is the front focal plane of lens L_0 and the back focal plane of lens L_1. Lens L_1 converts the incident spherical wave fronts of the scattered light to planar wave fronts which are diffracted by two parallel symmetric slits in plane 2. The diffracted light is focused by lens L_2 onto the detector plane 3, where Young's fringes are formed. When the scattering particle is a moving source, the light transmitted by the two slits is differentially shifted in frequency, forming moving interference fringes in plane 3 in analogy with the motion of real fringes at the probe volume in the dual-Doppler case when frequency offset is employed. The direction of motion of the fringes is obtained by the temporal correlation of the photoelectric signal from two, spatially offset detectors placed in plane 3.

If we express the complex amplitude of the laser beam in plane 1 as $E_1 = A(x)$, the optical field transmitted by plane 1 can be expressed as

$$E_{1t} = A(x)[1 - G(x - l)], \tag{1}$$

where $G(x)$ represents the blocking function of the scatterer and $l = \mathbf{v} \cdot \mathbf{n}t$ represents the displacement of the scatterer normal to the optical axis from an arbitrary reference. The incident field on plane 2 is then the Fourier transform of E_{1t}, or

$$E_2 = \mathcal{F}\{A(x) \cdot [1 - G(x - l)]\} \tag{2}$$

$$= [a(y) - a(y) * g(y - l)], \tag{3}$$

where we have adopted a convention that functions represented by lowercase and uppercase letters, e.g., $a(x)$ and $A(x)$ are mutual Fourier transforms. The asterisk represents a convolution. In the following analysis, we shall disregard constant multipliers, lumping them in a series C_i.

The field transmitted by the double slits in plane 2 can be written as

$$E_{2t} = C_1[a(y) - a(y) * g(y - l)] \cdot h(y), \tag{4}$$

where $h(y)$ represents the pupil function of the slits such that

$$h(y) = 1 \begin{cases} -\Delta - \delta \le y \le -\Delta + \delta, \\ \Delta - \delta \le y \le \Delta + \delta, \end{cases}$$
$$= 0 \text{ otherwise.} \tag{5}$$

This represents two slits 2Δ apart on center and 2δ wide.

The field in plane 3 is the transform of Eq. (4), i.e.,

$$E_3 = C_2 \mathcal{F}[a(y) - a(y) * g(y - l)] * H(z),$$

or

$$= C_2 [A(z) - A(z)\, G(z + lf_2/f_1)] * H(z), \tag{6}$$

the factor $f_2/f_1 = -M$ is the magnification of lens system L_1 and L_2 and can be obtained following the transforms in an orderly way (see Ref. 7). Next we make the reasonable assumption that $\Delta \gg w$, where w represents the diameter of the laser beam in plane 2, so that no part of the direct beam is transmitted and have, from Eq. (6),

$$E_3 = -C_2 [A(z)G(z - Ml)] * H(z). \tag{7}$$

Writing Eq. (7) explicitly,

$$E_3 = -C_2 \int_{-\infty}^{\infty} A(z')G(z' - Ml) \cdot H(z - z')dz'. \tag{8}$$

If $G(z)$ is a narrow function, i.e., $G(z) = 1$ for $|z| < \varepsilon$ and $= 0$ otherwise, where ε is small in comparison with the spatial period of $H(z)$, Eq. (8) can be reduced to

$$E_3(z) = C_3A(l)H(z - Ml). \tag{9}$$

Equation (9) is central to the scheme for obtaining the direction of motion of the scatterer. It is seen from Eq. (9) that the spatial distribution of the electric field depends only on the detection optics and is simply the diffracted field of the receiver aperture, centered at the geometric image of the scatterer, with an amplitude proportional to that seen by the scatterer in plane 1. Since $l = \mathbf{v} \cdot \mathbf{n}t$, it is also implied that regardless of the shape of $H(z)$, provided that $H(z)$ is a slowly varying function compared with $G(z)$, the electric field forms a traveling wave at plane 3. The direction of travel is indicated by the sign of $(\mathbf{v} \cdot \mathbf{n})$.

It is thus possible to choose one of a large family of functions $H(z)$ [hence $h(y)$], which must only satisfy the one criterion stated above, and which result in a traveling wave electric field at the detector surface.

The function $h(y)$ we chose for this illustrative example was the double slit for which

$$H(z) = \frac{4f_2}{ikz} \sin(k\delta z/f_2) \sin(k\Delta z/f_2), \tag{10}$$

where $k = 2\pi/\lambda$ is the wave number. Substituting in Eq. (9),

$$E_3(z) = C_4 \frac{A(l)}{k(z - Ml)} \sin \frac{k\Delta(z - Ml)}{f_2} \sin \frac{k\delta(z - Ml)}{f_2}. \tag{11}$$

The implied constraint on the width of the function $G(x)$ now becomes

$$\varepsilon \ll f_1/k\Delta \text{ and } \varepsilon \ll f_1/k\delta, \tag{12}$$

making the sine functions in Eq. (11) slowly varying compared with $G(z - Ml)$.

The intensity distribution at the detector plane is, from Eq. (11),

$$I_3(z) = E_3E_3^* = C_5 \frac{A^2(l)}{(z - Ml)^2} \sin^2 \frac{k\delta}{f_2}(z - Ml) \sin^2 \frac{k\Delta}{f_2}(z - Ml). \tag{13}$$

Using a well-known trigonometric identity, Eq. (13) reduces to

$$I_3(z) = C_5 \frac{A^2(l)}{4(z - Ml)^2} \left[\cos \frac{k(\Delta + \delta)}{f_2}(z - Ml) - \cos \frac{k(\Delta - \delta)}{f_2}(z - Ml)\right]^2, \tag{14}$$

$$I_3(z) = C_6 \frac{A^2(l)}{(z - Ml)^2} \left[1 - \tfrac{1}{2} \cos \frac{2k(\Delta + \delta)}{f_2}(z - Ml) - \tfrac{1}{2} \cos \frac{2k(\Delta - \delta)}{f_2}(z - Ml) - \cos \frac{2\Delta k}{f_2}(z - Ml) - \cos \frac{2\delta k}{f_2}(z - Ml)\right] \cdot \tag{15}$$

Equation (15) represents the optical intensity distribution on the detector plane. The first term, unity, in the square bracket corresponds to a nearly uniform illumination across the detector surface. This term describes the pedestal and is of no further interest. Recalling now that $l = \mathbf{v} \cdot \mathbf{n}t$, the second, third, and fourth terms represent spatially traveling waves of center frequency

$$f_c = 2\mathbf{v} \cdot \mathbf{n}\Delta/\lambda f_1 \tag{16}$$

and a bandwidth

$$B = 2\mathbf{v} \cdot \mathbf{n} \cdot \delta/\lambda f_1. \tag{17}$$

These terms express the beating between light transmitted by the two slits and hence are the signal terms. The fifth term represents the self-beating of light transmitted by a single slit. We recognize Eq. (16) to be identical with the Doppler frequency in the dual Doppler mode, although, due to the Fraunhofer approximation of this model, the center frequency [eq. (16)] varies as $2\mathbf{v} \cdot \mathbf{n}\Delta/\lambda f_1$ rather than the exact form

$$f_c = \frac{2\mathbf{v} \cdot \mathbf{n}\Delta}{\lambda\sqrt{f_1^2 + \Delta^2}} \cdot$$

The spatial component in Eq. (15), i.e., $(z - Ml)$ represents left or right (up or down) traveling waves depending on the sign of $l = \mathbf{v} \cdot \mathbf{n}t$. It is this relation between the direction of motion of the scatterer and the direction of motion of the fringe train which is exploited to obtain the direction of flow without frequency biasing. It is of historical interest to note that previous Fourier formulations such as that of Rudd[6] precluded the examination of spatial structure in the optical field at the detector surface since a single photodetector was used. On the other hand, Ogiwara[4] used a linear differential diode array to generate a periodic spatial response function of the detector but only for the removal of the Doppler pedestal. It is straightforward to show Ogiwara's result from Eq. (15). If the spatial responsivity of the detector is periodic with spatial frequency equal to $K = 2k\Delta/f_2$, the photocurrent can be expressed as

$$I(t) = C_6 \int_{-\infty}^{\infty} A^2(l)|H(z - Ml)|^2 \sum_{n=1}^{\infty} B_n \cos(nKz + \phi_n)dz, \tag{18}$$

where $\omega = 4\pi(\mathbf{v} \cdot \mathbf{n})\Delta/\lambda f_1$, and $\Delta \gg \delta$ is assumed, and the series $\Sigma_{n=1}^{\infty} B_n \cos(nKz + \phi_n)$ is the Fourier representation of detector responsivity. Thus only spatial frequencies of $nK/2\pi$ are detected. Lower frequencies represented by terms 1 and 5 in Eq. (15) produce no photocurrent, and the pure zero-mean Doppler current is obtained.

III. Determination of Direction

The determination of the flow direction is achieved if the direction of travel of the fringe train on the detector plane is obtained. This amounts to determining the algebraic sign of $\mathbf{v} \cdot \mathbf{n}$. To accomplish this two spatially separated samples of the fringe train are required, and a correlation procedure is employed. If the two diode detectors are $\frac{1}{4}$ fringe apart, the photocurrents resulting from the two diodes will be $\pm\frac{1}{4}$ Doppler period apart. One needs only to compare the two correlations to obtain the direction of motion. The direction is positive or negative according to $\phi(\pi/2) - \phi(-\pi/2) \gtrless 0$, where the function $\phi(\tau)$ is $\phi(\tau) = \langle I_1(t - \tau)I_2(t)\rangle$.

In this work, we use two linear differential diode arrays similar to that of Ogiwara. The two photocurrents are then pedestal-removed zero-mean processes. The currents can be written, from Eq. (18), as

$$I_1(t) = C_7A^2(l) \int_{-\infty}^{\infty} |H(z - Ml)|^2B_1 \cos(Kz + \phi_1)dz,$$

$$I_2(t) = C_7A^2(l) \int_{-\infty}^{\infty} |H(z - Ml)|^2B_1 \cos(Kz + \phi_1 + \pi/2)dz. \tag{19}$$

The direction of flow is obtained from running two delayed correlations $\phi(\pm\pi/2)$ and differencing as explained.

The choice of differential diode arrays is indicated by yet another consideration: Signal strength. To receive the full power of the optical signal, the entire fringe train must be detected at all times. Since noise due to photodetectors increases as the square root of number of detectors, whereas the signal strength increases linearly, a net gain in SNR is realized.

IV. Effect of Noise on Direction Determination

Uncertainty in the determination of direction can result from contributions to the correlation function $\phi(\tau)$ from the shot noise of the optical signal. It is straightforward to show (Ref. 8) that the autocorrelation function of bandlimited white noise is expressed as $\phi_n(\tau) = \sigma^2 \operatorname{sinc}(2\pi f_m \cdot \tau)$, where σ^2 is the total noise power, and f_m is the upper frequency limit. Thus, the statistical uncertainty in $\phi(\tau)$ at $\tau = \frac{1}{4}f_c$ is the autocorrelation function of the noise itself given by

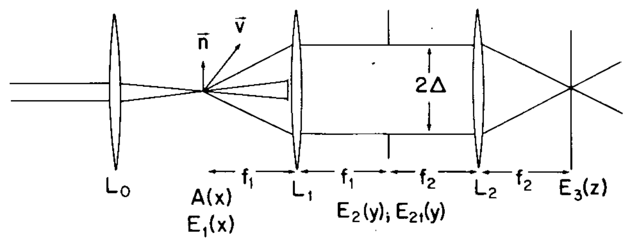

Fig. 2. Schematic of directional LDV with large aperture. The use of a wider aperture improves SNR. A square aperture allows two-axis velocity measurement.

$$\phi_n\left(\frac{1}{4f}\right) = \sigma^2 \operatorname{sinc}\left(\frac{\pi f_m}{2f_c}\right) \cdot$$

If ξ is the noise-to-signal power ratio per unit bandwidth

$$\frac{\phi_n(\tau)}{\phi(\tau)} = \xi f_m \operatorname{sinc}\left(\frac{\pi}{2}\frac{f_m}{f_c}\right) \text{ or } = \xi \sin\left(\frac{\pi f_m}{2f_c}\right), \tag{20}$$

which indicates the benefit of a bandwidth approximately twice the Doppler frequency, and estimates an upperbound on error if the noise power per unit bandwidth is known. This noise power is eI/η, where e is the electronic charge, I is the mean current, and η is the detector quantum efficiency.

V. Discussion

The existence of spatial structure in the detector plane thus makes possible the determination of the direction of motion and removal of pedestal. The only restriction in the model presented is that the particle size ϵ be smaller than $\lambda f_1/\Delta$. This quantity will be recognized by workers familiar with dual LDVs as the spacing of fringes at the measurement volume (in this case virtual fringes replace the real fringes of a dual-Doppler LDV). Thus, the requirement is that the scatterer be smaller than the virtual fringe spacing.

The double slit example chosen here is one of the class of masks which are allowed by such a method. In fact, one obviously reduces signal strength by the use of slits, and other masking functions $h(y)$ should be considered. A particularly attractive one is shown in Fig. 2:

$$h(y) = 1 \qquad -\Delta < y < \Delta \qquad = 0 \text{ otherwise.}$$

Thus a rectangular wide slit is suggested. The particle size restriction can be seen to remain $\varepsilon \ll \lambda f_1/\Delta$. As before, the spatial period required for the detector array is $K = 2k\Delta/f_2$. $H(z)$ in this case is the sinc function.

Finally, we observe that violation of the condition on the size of the scatterer only alters the shape of the optical field distribution; the traveling wave nature is not destroyed. Thus a broader application of this principle in removing Doppler directional ambiguity appears possible. Experimental implementation will be described in a complementary paper.

VI. Summary

It is demonstrated that the spatial distribution of the optical field at the detector plane of an LDV represents a traveling wave. If the scattering center is sufficiently small, i.e., equivalently smaller than the virtual fringe spacing of the detector optics at the measurement plane, the optical field at the detector is simply the Fourier transform of the detector pupil function. The direction of the traveling wave is determined by sampling at two points and differencing two delayed correlations. The use of two differential diode arrays simultaneously eliminates the pedestal.

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