Continental Shelf Research, Vol. 2, No. 1, pp. 21 to 33, 1983.
Printed in Great Britain.

0278–4343/83 $3.00 + 0.00

Eigenanalysis of recent United States sea levels

D. G. AUBREY* and K. O. EMERY*

(*Received* 21 *September* 1982; *in revised form* 28 *December* 1982; *accepted* 6 *January* 1983)

Abstract—Spatial and temporal patterns of recent sea-level rise along the United States coastline have been examined to ascertain rates of rise, and possible causes for high-frequency fluctuations in sea level. Eigenanalysis identified several distinct coastal compartments within each of which sea-level behavior is consistent. The United States east coast has three of these compartments: one north of Cape Cod, where sea-level rise increases with distance to the north; one between Cape Cod and Cape Hatteras where sea-level rise increases to the south; and the third from Cape Hatteras south to Pensacola, where sea-level rise decreases to the south. The western gulf coast represents another compartment (poorly sampled in this study), where subsidence is partly due to compaction. The final compartment is along the United States west coast, where poor spatial sampling produces a highly spatially variable sea-level record that has some temporal uniformity. Spectral analysis shows a dominant time scale of six years for sea-level variability, with different coastal compartments responding relatively in or out of phase. No evidence for increased rates of sea-level rise over the past 10 years was found. This objective statistical technique is a valuable tool for identifying spatial and temporal sea-level trends in the United States. It may later prove useful for identifying elusive world-wide trends of sea level, related to glacial melting, glacial rebound, tectonism, and volcanic activity.

INTRODUCTION

CHANGES of relative sea level occurred throughout the entire length of the geological record, but during historical time they have had progressively larger effects upon man and his activities, especially in coastal belts of industrial nations. Difficulties experienced by Phoenician, Greek, and Roman builders with submergence or emergence of harbors, quays, buildings, fish ponds, and salt evaporation ponds have parallels in modern times. Most modern effects are local and due to volcanic or tectonic disturbances and withdrawal of interstitial fluids. Widespread rapid eustatic sea-level rise occurred during the main melting of the latest ice sheets prior to about 5000 years ago, and renewed rapid rise may occur from predicted melting of the West Antarctica ice sheet because of rise in polar temperatures produced by increased blanketing by carbon dioxide from burning of fossil fuels. This new rise may total as much as 5 m in 40 years (e.g., MERCER, 1978). Even a few decimeters rise in sea level is expected to produce serious erosion of beaches and seacliffs accompanied by destruction of buildings and roads built upon them. Because the effects of such a rise can be far greater than experienced by modern engineers, an effort must be made to identify (if possible) and document its early stages.

Radiometric (chiefly ^{14}C) ages of plants and animals that lived only in intertidal or shallow marine waters, but whose remains became submerged by rising sea level, show a post-glacial rise in excess of 100 m (e.g., CURRAY, 1960; MILLIMAN and EMERY, 1968). Regional

---

* Woods Hole Oceanographic Institution, Woods Hole, MA 02543, U.S.A.

differences in relative sea-level change along eastern United States during the past 5000 years were examined by REDFIELD (1967). However, the precision of the ^{14}C method ordinarily is of the order of half a meter and a thousand years, clearly too coarse for the current problem.

A better method may be the use of tide-gauge measurements, whereby records at a given station are averaged hourly for a full year to obtain mean annual sea level at that station. Plots of mean annual relative sea levels for many years may show progressive changes at that station indicative of long-term change of sea level or of the ground surface to which the recording apparatus is fixed. On the other hand, it may show periodic changes due to changes of annual river runoff that controls local sea level; MEADE and EMERY (1971) found that 7 to 21% of the variability in groups of east-coast United States stations could be attributed to variation in runoff—especially important because most tide-gauge stations are in harbors at the mouths of large rivers. This runoff correction cannot be applied routinely, because runoff data are not compiled in a simple-to-use form, and in most countries it is not even available. Another limitation to tide-gauge data is the poor world-wide distribution pattern of stations. Three greatest densities are in industrialized nations: United States (where west-coast stations are subject to tectonism associated with plate movements), Europe (where northern stations are influenced by crustal rebound after melting of the Scandinavian ice sheet), and Japan (where both volcanism and plate tectonics combine to make erratic records). Lesser developed nations in Africa, South America, Central America, and southern Asia have only a few tide-gauge stations, most of which span short terms, have many gaps in records, or have been discontinued.

There are roughly 800 tide-gauge stations in the world (e.g., LENNON, 1975 to 1978). When we eliminate those that have ceased operations, those whose results are mostly secret (Soviet Union and China), and those with records too short or too interrupted for reliable use, a total of only about 250 stations remain. Of these, 92% are in the northern hemisphere, where there are further reductions in number because of known movements of coasts by post-glacial uplift, volcanism, and tectonism. In addition to known disturbances of some coasts, other previously unknown or poorly known disturbances may affect other coasts. This means that a series of records in any single region cannot be taken as indicative of eustatic change in sea level for the world ocean. For this reason EMERY (1980) chose to plot sea-level changes at all reliably recorded stations on a histogram from which the *median* value was taken as the most reasonable change for the world-ocean level. GORNITZ, LEBEDEFF and HANSEN (1982) and BARNETT (1983), instead, chose to eliminate unstable stations subjectively and to divide the world ocean into 14 and 11 units (respectively), averaging the results from remaining stations in each unit, and then obtaining a mean for all units to serve as the value for the world ocean. This approach reduces the weighting by the many tide-gauge records in the industrialized northern-hemisphere nations, but it equates these many well-recorded stations with perhaps a single (relatively poor) station in each of several areal units. Perhaps a better way of comparing and grouping relative sea-level changes in different regions can be found.

There also is the problem of how to determine the trends of mean annual sea levels for each station. Nearly all investigators have chosen to fit least-squares straight lines to the data points. In his long series of articles on United States sea levels, HICKS (1978; HICKS and SHOFNOS, 1965; HICKS and CROSBY, 1974) obtained generally increasing slopes according to the length of the record (at successive dates of compilation for the same station). This suggests that the rate of sea-level rise increased with time. Emery found a similar progressive increase for world-wide rates for the time span between start of the station record and 1975, the time span between 1966 and 1975, and the time span between 1970 and 1974. An uncertainty exists of whether this recent increase is real or whether it merely reflects some sort of climatic cycle imposed upon sea level. One possible remedy is to fit polynomial (particularly binomial) curves to the data points for each station. By this means, S. D. HICKS and L. E. HICKMAN (personal communication) found that of 15 long-term United States tide-gauge stations, best fit for five is obtained by accelerating rates, two by constant rates, and eight by decelerating rates—divided about equally between east and west coasts. Another method that shows promise and is more objective than others for both selection and fitting of curves to data points is that of eigenfunction analysis; we test it here for the United States stations before expanding it to the world ocean.

METHODS

Two specific types of information are contained in sea-level records. First, the relative rates of sea-level rise, including acceleration or deceleration of this rise, are important for coastal stability considerations. Second, the spatial and temporal structure of relative sea-level rise will help clarify causes and time scales of variation in sea levels.

Linear and non-linear regression techniques used previously do not provide the second type of information, nor do they determine coherent modes of sea-level change to minimize local aberrations in sea level. Instead, they impose a subjective shape to sea-level curves, and optimally fit a local record to that shape. We use here an objective method to determine uncorrelated modes of sea-level change along the entire United States coastline: eigenanalysis.

Eigenanalysis is a well-known widely-used objective technique for determining dominant modes of variation in data sets. It has early roots in the behavioral and social sciences, and is known variously as principal component analysis, factor analysis, empirical eigenfunction analysis, or empirical orthogonal function analysis. The technique has been in common use for weather and climate prediction (LORENZ, 1959; HAYDEN and SMITH, 1982), for studies of ocean–atmosphere interaction (DAVIS, 1976), and for analysis of beach profile and surface wave data (WINANT, INMAN and NORDSTROM, 1975; AUBREY, 1979, 1980). The purpose of eigenanalysis is to separate a data set into orthogonal spatial and temporal modes which most efficiently describe the variability of that data set (no other orthogonal functions can more efficiently represent that same data set). The result is a concise description of the spatial and temporal structure of variability in that data set.

Mathematically, a data set $\eta(x, t)$ can be decomposed into spatial and temporal functions:

$$\eta(x, t) = \sum_{k=1}^{N} C_k(t) e_k(x) (\lambda_k n_x n_t)^{1/2}, \tag{1}$$

where $\eta(x, t)$ is a spatial grid sampled through time, the $C_k(t)$ represent temporal eigenfunctions, $e_k(x)$ represent spatial eigenfunctions, λ_k are the eigenvalues, n_x is the number of spatial points, n_t is the number of temporal points, and N is the lesser of n_x and n_t. Spatial eigenfunctions are determined from the following matrix operation:

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{e} = 0,$$

where (2)

$$\mathbf{A} = \frac{1}{n_x n_t} \eta\eta^T.$$

The superscript T refers to the matrix transpose operator, $\mathbf{I}$ is the identity matrix, η is an (n_x, n_t) matrix of data, and $\mathbf{A}$ has size (n_x, n_x). Temporal eigenfunctions are determined from the matrix equation:

$$(\mathbf{B} - \lambda\mathbf{I})\mathbf{C} = 0,$$

where (3)

$$\mathbf{B} = \frac{1}{n_x n_t} \boldsymbol{\eta}^T\boldsymbol{\eta}.$$

B is a (n_t, n_t) matrix. The λ's are identical for equally ranked spatial or temporal eigenfunctions. Eigenfunctions are ranked according to their eigenvalues; the first eigenfunction has the largest eigenvalue, the second eigenfunction has the next largest eigenvalue, and so on. The eigenfunctions obey a least-squares criterion: the first eigenfunction best describes the data in a least-square sense; the second eigenfunction best describes the residual of the data in a least-squares sense, and so on. All eigenfunctions are orthonormal (or uncorrelated):

$$\mathbf{C}\mathbf{C}^T = \mathbf{I}$$

$$\mathbf{e}\mathbf{e}^T = \mathbf{I}.$$

Eigenfunction equations (2) and (3) are solved numerically (see, e.g. WILKINSON and REINSCH, 1971), after matrices **A** and **B** are calculated. Operationally, once either set of eigenfunctions is determined numerically, the second set can be determined from the inner product of the eigenfunction and the data matrix. Once eigenfunctions are calculated, the original time series (data set) can be reconstructed completely using equation (1). If the first 'm' eigenvalues dominate all other eigenvalues (where $m < N$), a filtered data set ($\eta'(x, t)$) can be generated using (1), by summing only up to $k = m$. This new filtered data set ($\eta'(x, t)$) can then be used to represent the 'predictable' or information-rich part of the original signal. In the following analysis, we generally obtain a useful $\eta'(x, t)$ by letting $m = 2$. A more complete discussion of statistical methods for selecting information-rich eigenvectors is presented in PREISENDORFER, ZWIERS and BARNETT (1981).

Since either the $C_k(t)$ or $e_k(x)$ eigenfunctions can be viewed as weighting functions, we use the convention of presenting data as normalized $C_k(t)$ for temporal variability, and physical quantities $e_k(x)a_k$ with units of meters for spatial variability, where

$$a_k = (\lambda_k n_x n_t)^{1/2}.$$

Before the eigenfunctions are determined, each spatial data set (i.e., for a particular station) has its mean removed.

Advantages of the eigenfunction technique include the following:

(1) The technique is objective, and does not impose a pre-determined form to the data, although the resulting functions can later be best-fit to any subjective curve.

(2) It provides an objective means of ranking uncorrelated modes of variability to eliminate weak signals or 'noise' from the data.

(3) It facilitates interpretation of spatial and temporal variability. Spectral analysis can describe dominant frequency or wavenumber components. The temporal trends can be used as input to predictive models of sea-level variability.

(4) It produces modes of variability which are uncorrelated with one another.

(5) It represents modes of change which are coherent in both space and time in the most efficient way possible.

A disadvantage of the technique is that spatial patterns (e_k) need have no physical analogue or reflect any obvious physical pattern. This is especially true if there are no dominate eigenvalues (i.e., eigenvalues are degenerate).

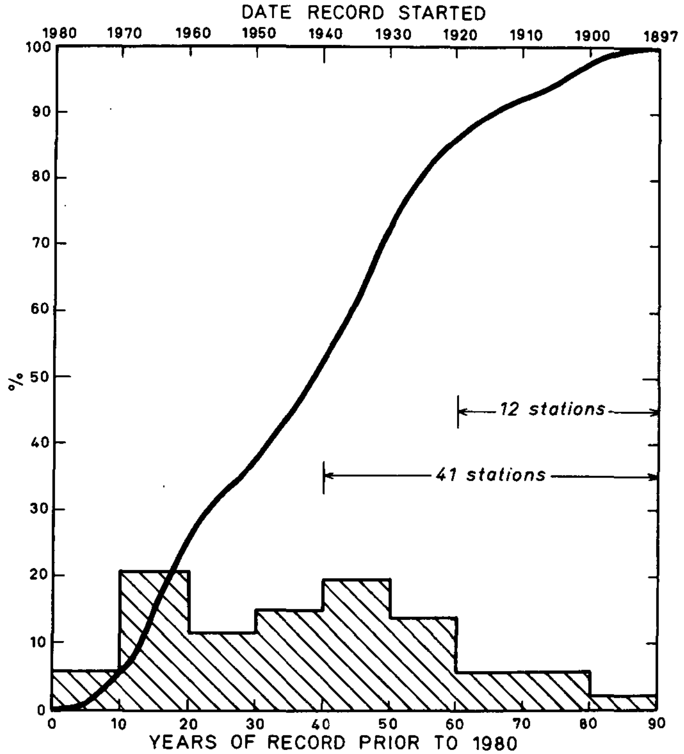

Fig. 1. Percentage distribution of starting dates and numbers of years of continuous record prior to 1980 of 87 United States-operated tide-gauge stations from which mean annual sea levels are available.

The data source for this study is magnetic tape from the National Ocean Survey (NOAA) supplied by L. E. Hickman of the Tidal Analysis Section in early July 1982. This tape contains mean annual sea levels for United States tide-gauge stations, including Honolulu (Hawaii) and Cristobal (Panama) through 1979. Eighty-seven stations are essentially uninterrupted through the year 1979. Plots of length of record by histogram and cumulative curve (Fig. 1) reveal a median record length prior to 1980 of 42 years for these stations.

RESULTS AND DISCUSSION

Analysis of United States sea-level data was performed using two different time scales. First, to obtain a general view of the results to be expected from eigenanalysis, we analyzed 12 tide-gauge stations from all United States coasts having a continuous 61-year record prior to 1980. The first temporal function revealed a jagged year-by-year rise of sea level during the 61 years (Fig. 2), such that a best-fit straight line through the data array indicates a mean relative rise of sea level ranging from +6.3 mm y^{-1} (Galveston) to −0.26 mm y^{-1} (at Ketchikan), with an average of +1.4 mm y^{-1} on the west coast, and +1.3 mm y^{-1} on the east coast (Fig. 3). The second temporal function showed no significant mean trend. Our spatial analysis showed that the stations belong to several different distinct populations, whose

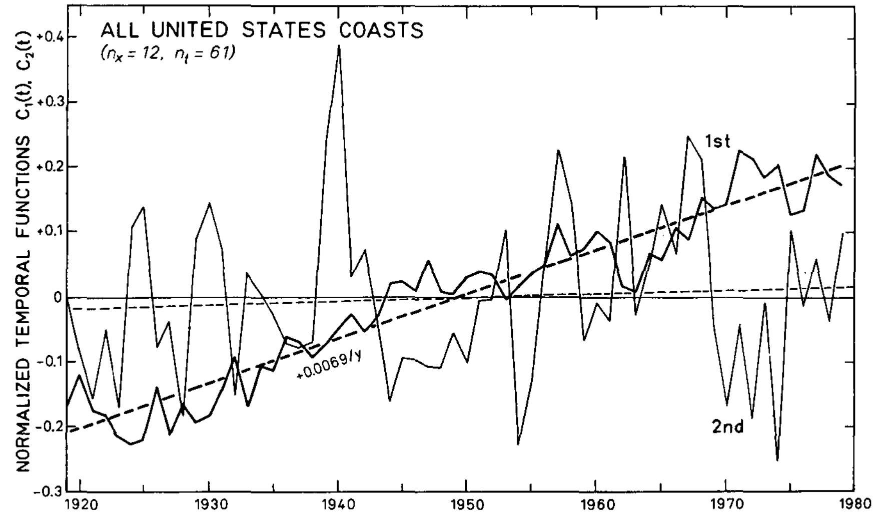

Fig. 2. Temporal eigenfunctions for 12 stations having records of 61 years. First function accounts for 83% of total variation, and second function for only 6%. Linear trend for the first function must be weighted by corresponding spatial value to obtain local rate of sea-level rise.

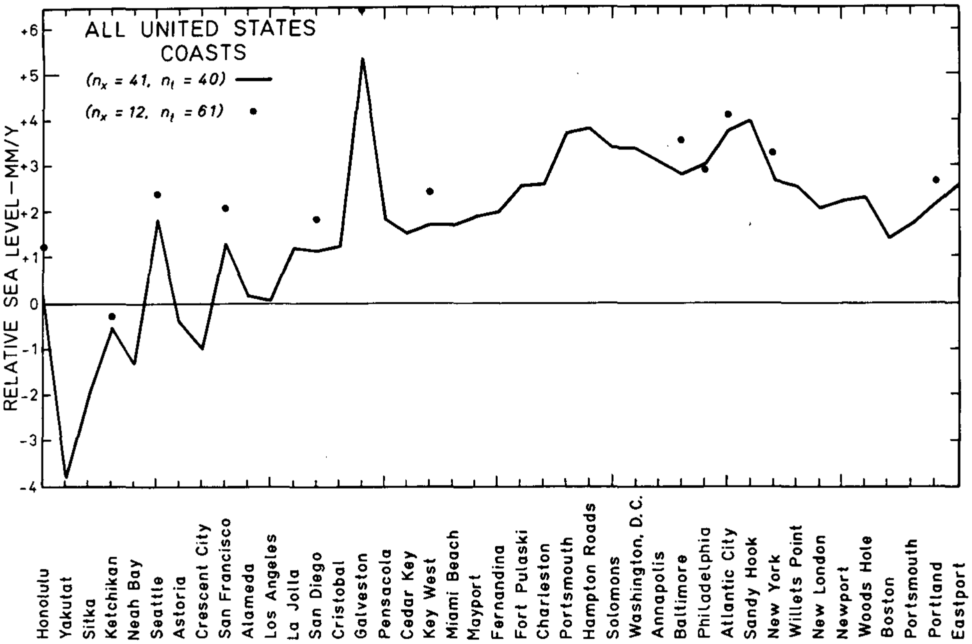

Fig. 3. Relative change of sea level at each of 12 61-year stations from Fig. 2, and at each of 41 40-year stations computed from eigenfunctions of Figs 4 and 5.

characteristics were further explored using additional stations having a shorter 40-year time span.

Second, in a trade-off between length of records and good spatial coverage, eigenanalysis for all United States coasts was repeated on 41 40-year stations, distributed around the United States coastline. Spatial functions (Fig. 4) reveal different patterns for west and east coasts. For the first function, spatial patterns for the west coast (including Honolulu and Alaska) show both positive and negative weightings (corresponding to positive and negative

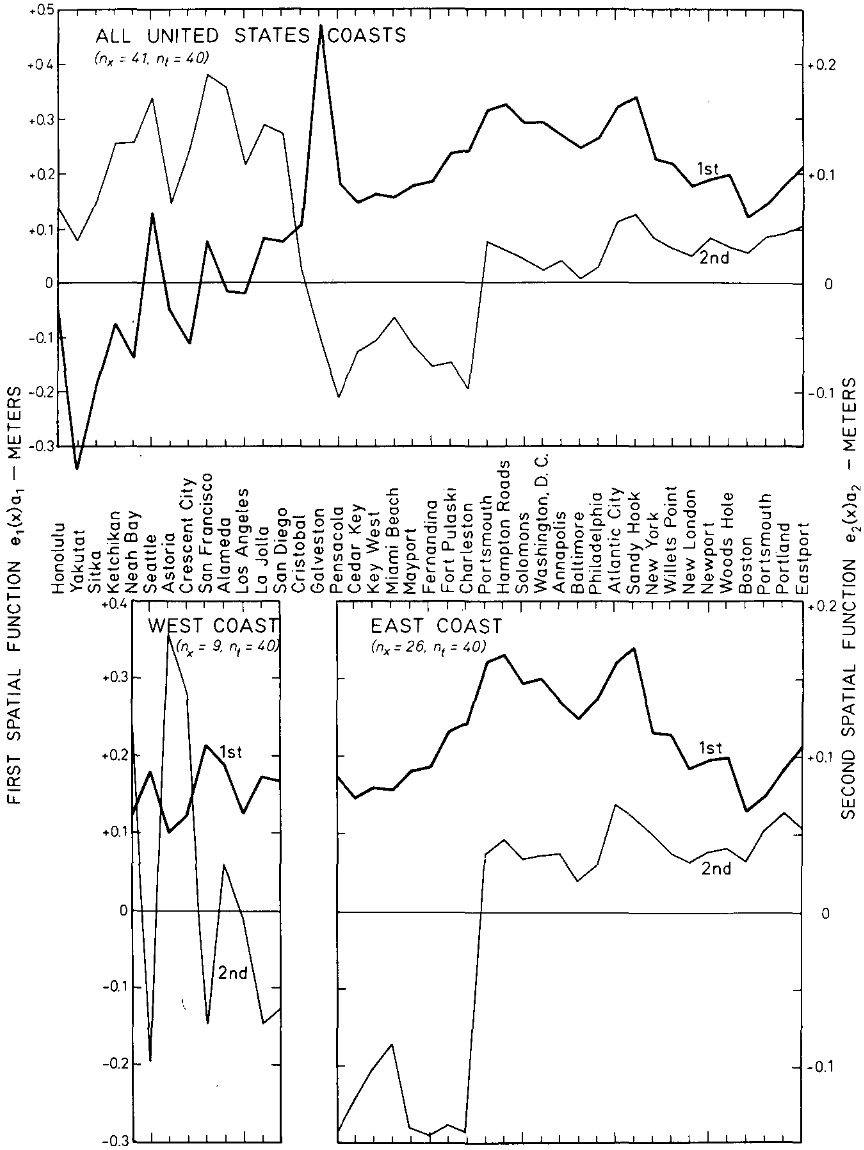

Fig. 4. Spatial eigenfunctions for 41 stations having records of 40 years, with similar plots for 10 west-coast stations and 26 east-coast stations. First functions account for 65, 53, and 78% of total variations, and second functions for 10, 26, and 10%, respectively.

sea-level rise), with no obvious pattern. Gulf coast stations are uniform and positive, while Atlantic coast stations show two regions (both positive), one with higher weightings than the other. For the second function, west-coast stations are uniformly positive, all gulf coast and southern east-coast stations are negative, while northeast coast stations have low positive weightings. These were further investigated and confirmed using separate spatial functions for west coast (Neah Bay to San Diego) and east coast (Pensacola to Eastport), omitting the more isolated or irregular stations of Honolulu, Yakutat, Sitka, Ketchikan, Cristobal, and Galveston. Similar relative spatial patterns emerged (Fig. 4).

Next, the temporal functions were computed for the same three station subsets—all United States coasts, west coast, and east coast (Fig. 5). Mean rates of relative rise in sea level are 1.7, –0.3, and 2.5 mm y^{-1}, respectively. R-tests exceeded 99.9% (except for west coast), and no improvement was obtained by trials of polynomial curves fitted to the data for the entire United States.

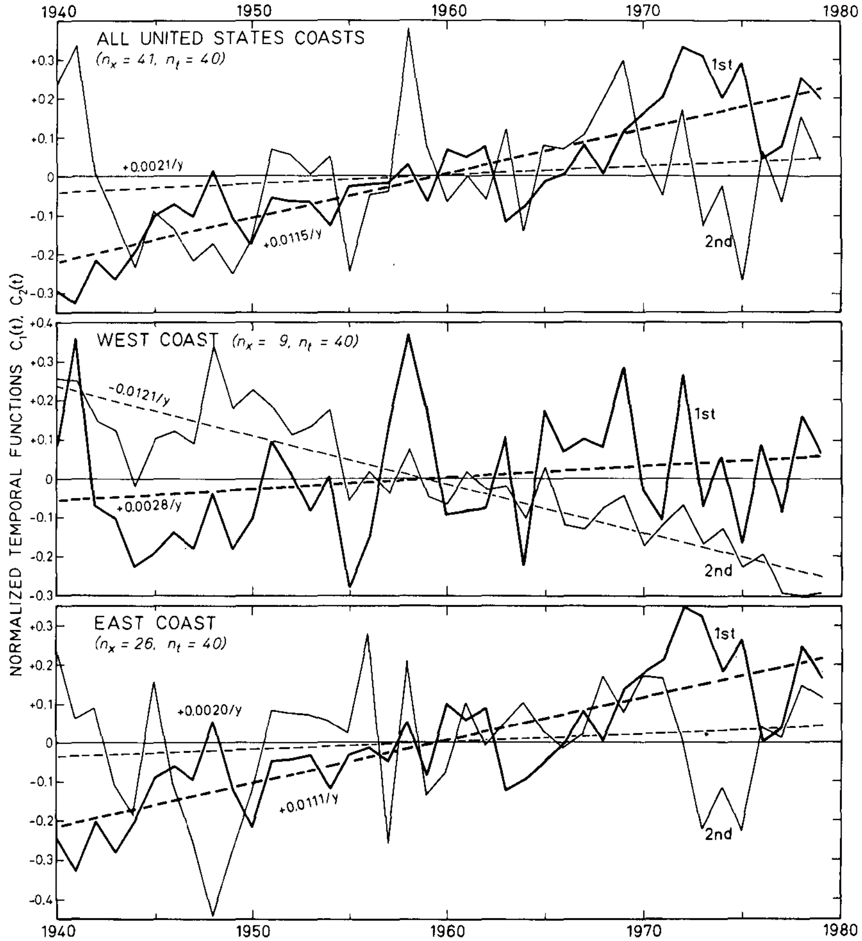

Fig. 5. Temporal eigenfunctions for same stations as Fig. 4. First functions account for 65, 53, and 78% of total variations, and second functions for 10, 26, and 10%, respectively. Linear trend for the first function must be weighted by corresponding spatial value to obtain local rate of sea-level rise.

By combining linear trends from the first and second temporal eigenfunctions (Fig. 5) for the 40-year span, we obtained mean rates of relative sea-level rise for each separate station of the United States (Fig. 3). These rates confirm the station grouping that is evident on spatial functions for the spatially limited 61-year data set. Mean sea level at Yakutat is -3.8 mm y^{-1}—in accordance with known tectonic uplift of 14 m during an earthquake in 1899 (Tarr and Martin, 1912) with presumed slower continuation to the present. Galveston, on the other hand, is indicated as having a mean rise in relative sea level of $+5.3$ mm y^{-1}, supporting the general belief in ground subsidence there due to withdrawal of petroleum, perhaps supplemented by compaction of Quaternary sediments in the region of the tide gauge. The relative change in other stations used for west-coast and east-coast computations of Fig. 4 exhibit a smaller range of variation. Especially interesting are the variations on the east coast.

The temporal functions based upon 61 years (Fig. 2) and 40 years (Fig. 5) appear to include consistent frequencies, so a spectral analysis was made. This analysis (Fig. 6) shows similarities in peak frequencies between spectra for both sets of year spans in both first and second functions. Aside from a very long period on the order of the duration of record, the dominant period is about 6 years, with others at about 3.2, 2.9, and 2.3 years, the significance of which is presently unknown but may be related to atmospheric or oceanic climatic cycles. The very long period merely reflects the long-term general rise of sea level.

Subunits of the east coast that are suggested by eigenfunctions of Figs 3 to 5 were examined further. Those south of Cape Hatteras (Pensacola to Charleston) and those north of Cape Hatteras (Portsmouth, VA, to Eastport) proved to belong to different groups relative to

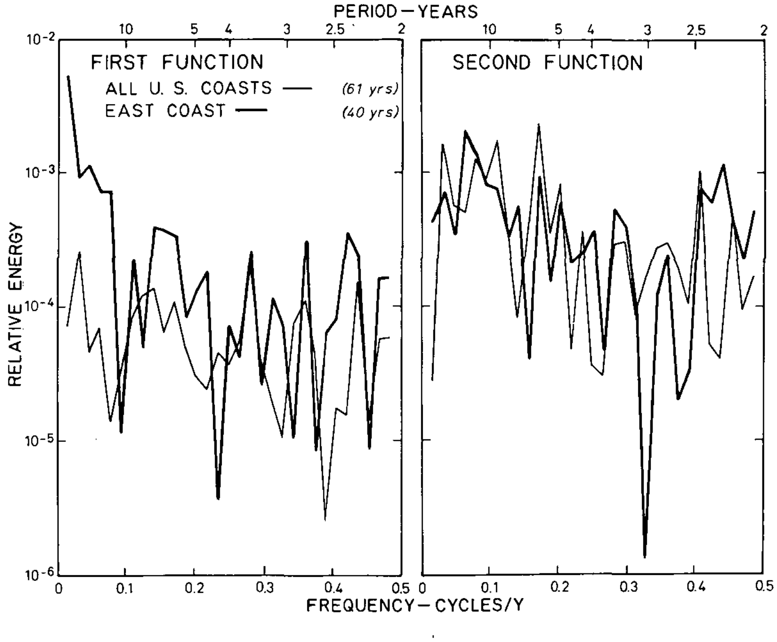

Fig. 6. Spectra of first and second eigenfunctions for 12 United States stations of 61 years and for 26 east-coast stations of 40 years. Estimates have two degrees of freedom. Energy units are relative.

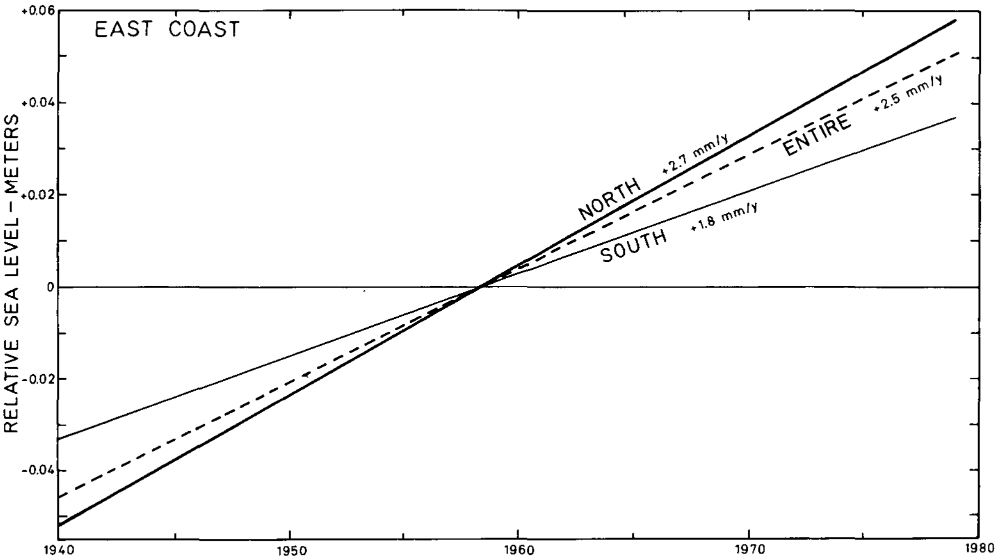

Fig. 7. Mean sea-level change for 40 years of south (Pensacola to Charleston), north (Portsmouth, VA, to Eastport), and for entire east coast (8, 18, and 26 stations, respectively), determined from eigenfunctions.

sea-level trends—+1.8 mm y^{-1} in the south and +2.7 mm y^{-1} in the north (Fig. 7). Values for individual stations were computed from first and second temporal functions and plotted in Fig. 8. Three different trends are indicated: decreasing rates with increasing distance south of Cape Hatteras, decreasing rates with increasing distance northeast of Cape Hatteras (as far as Cape Cod), and increasing rates northeast of Cape Cod. The trend northeast of Cape Cod

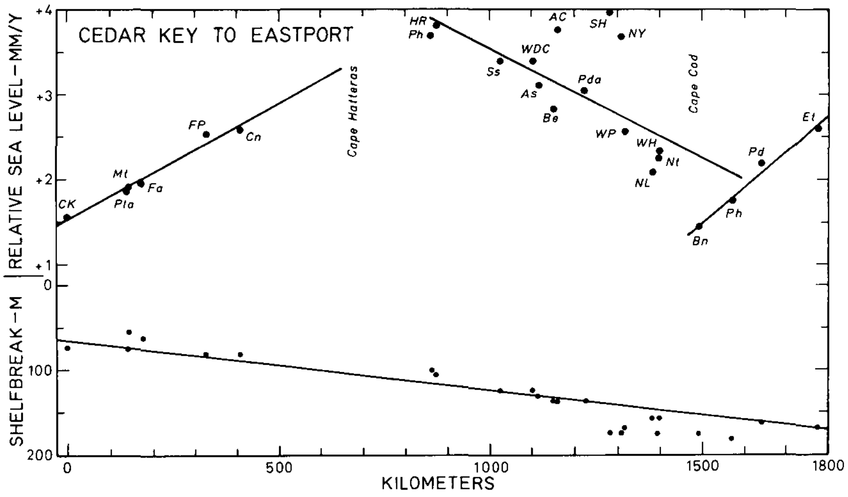

Fig. 8. Mean annual relative sea-level changes during 40-year record from Figs 3, 4, and 7 for east-coast stations. Regression lines denote three main segments of east coast having different sea-level trends; these trends are not reflected in depths to shelf break at points nearest each station.

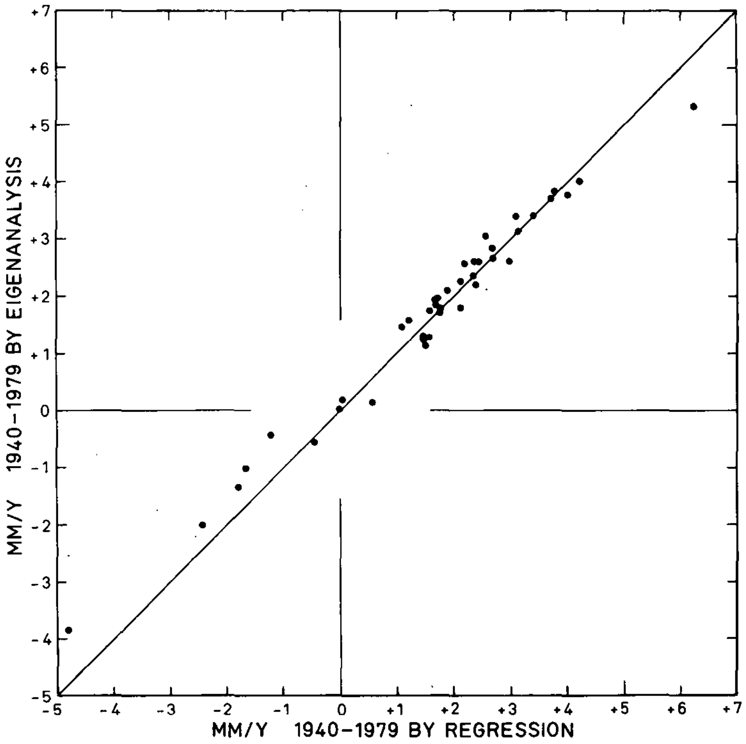

Fig. 9. Scatter diagram of 40-year sea-level rise estimates determined from eigenfunction analysis and by linear regression. Straight line is theoretical best fit.

may be associated with indirect effects of glacial rebound, but the high rates of relative sea-level rise on either side of Cape Hatteras were unanticipated. No known intersections of plate fracture zones or other structures have been recognized in that vicinity. Moreover, the depth of the shelf break along the entire coast seems to be unrelated to the sea-level rate changes during the past 40 years, implying that the latter may not be due to continuing long-term consistent plate deformation.

Finally, as a check on the accuracy of the eigenfunction representation of spatially coherent linear sea-level trends, the eigenfunction estimates were compared to simple linear regression estimates for each station for the same 40-year period of time (Fig. 9). The small amount of scatter about the theoretical 45° line of fit supports the use of eigenfunctions for inferring spatially coherent linear sea-level trends.

Finally, we checked the hypothesis that rate of sea-level rise is increasing through time. Keeping in mind the time-limited data set which makes difficult statistically valid computations, no evidence for increased rate of sea-level rise over the past 10 years was observed. Previous suggestions of such an increase in rate may have been based on an inadequate statistical sample. Eigenanalysis of all available world stations sampled through 1981 will help resolve the question of recent acceleration in sea levels.

CONCLUSIONS

Eigenanalysis of mean annual sea levels recorded at United States tide-gauge stations offers a means of estimating coherent average annual changes in relative sea level as well as providing a time variation for these spatially coherent trends. It also identifies groups of stations having similar sea-level records in an objective way that discounts occasional aberrant values that can affect fitting of regression lines to the data points for individual records. Specifically, the eigenanalyses showed that west-coast stations (including Honolulu and Alaska) are rather erratic—presumably because of land movements produced by plate tectonism or volcanism, or both. East-coast stations that include the eastern Gulf of Mexico are less erratic, but they still indicate the presence of three coastal sections having different trends of relative sea-level changes. Systematic variations of sea level occur along each coastal section with trends that are not paralleled by depths to shelf break or by any other obvious topographic or structural features. Instead, this spatial stratification may reflect oceanographic forcing (such as ocean temperature, sea-level pressure, Gulf Stream meanders). Recognition of the coastal sections was permitted only by the close spacing of reliable long-term tide-gauge stations along the east coast.

Similar trends were not recognized for the more widely spaced west-coast stations, and they seem unattainable for much of the rest of the world because of wide spacing of stations relative to size of tectonic units. For example, equally reliable long-term stations are rare in the southern hemisphere—only two in all of Africa, and three in South America. The method may be able to identify which of the many Scandinavian stations may be outside the area of glacial rebound for comparison with middle and south European stations. It also may serve to identify relatively stable stations in Japan, where many long-term stations exist, but many of which are erratic as though due to local volcanic deformation of the ground on which the stations are sited.

Simple averages of records from random or selected tide-gauge stations throughout the world are considered unreliable for identifying any world-ocean rise of eustatic sea level. Comparison of regression estimates of sea-level trends at a particular United States station for different record lengths shows that rate of sea-level rise is dependent on record length (since fluctuations are nearly as large as the net change). However, the similarities in spatial patterns of sea-level rise between the widely spaced 12 61-year stations and the 41 more closely spaced 40-year stations suggest that sparse sampling may be enough to represent energetic (important) sea-level patterns. This suggests a spatial Nyquist criterion, that the spatial sampling must be more dense than the wavelength of the dominant patterns of change.

Eigenanalysis may improve the reliability of world-wide sea-level estimates (BARNETT, 1983). The main drawback to this type of analysis is that it requires many stations having similar records and for the same time span. The advantages to this technique are considerable:

(1) Sea-level trends are representative of the data itself, not some preconceived notion of the linearity of the trend.

(2) The technique selects patterns which are the most energetic in the sense that they describe most of the mean-square value (or variance) in the data sets in a least-squares sense. This is useful for screening out noise from the data sets. It also enables one to stratify, or distinguish, areas having different sea-level response.

(3) The temporal information gives a time series useful in predictive models relating sea-level changes to oceanic and atmospheric climatic effects. It also provides information on time scales of sea-level variability.

(4) Resulting objective temporal patterns can then be best-fit to any subjective time trend (such as with linear or non-linear regression).

Acknowledgements—The first author was supported by NOAA, Office of Sea Grant under Grant No. NA80-AA-D-00077 (R/B-21) and by the U.S. Army Research Office, under Grant DAAG29-81-K-0004. G. P. LOHMANN and E. UCHUPI of this institution provided helpful comments on the manuscript. Woods Hole Oceanographic Institution Contribution No. 5232.

REFERENCES

AUBREY D. G. (1979) Seasonal patterns of onshore/offshore sediment movement. *Journal of Geophysical Research*, **84**, 6347–6354.

AUBREY D. G. (1983) The statistical prediction of beach changes in southern California. *Journal of Geophysical Research*, **85**, 3264–3276.

BARNETT T. P. (1982) Recent changes in sea level and their possible causes. *Climate Change*, **5**, 15–38.

CURRAY J. R. (1960) Sediments and history of Holocene transgression, continental shelf, northwest Gulf of Mexico. In: *Recent sediments, Northwest Gulf of Mexico*, F. P. SHEPARD, F. B. PHLEGER and Tj. H. VAN ANDEL, editors, American Association of Petroleum Geologists, pp. 221–266.

DAVIS R. E. (1976) Predictability of sea surface temperature and sea level pressure anomalies over the North Pacific Ocean. *Journal of Physical Oceanography*, **6**, 249–266.

EMERY K. O. (1980) Relative sea levels from tide-gauge records. *Proceedings of the National Academy of Sciences*, **77**, 6968–6972.

GORNITZ V., S. LEBEDEFF and J. HANSEN (1982) Global sea level trend in the past century. *Science*, **215**, 1611–1614.

HAYDEN B. P. and W. SMITH (1982) Season-to-season cyclone frequency prediction. *Monthly Weather Review*, **110**, 239–253.

HICKS S. D. (1978) An average geopotential sea level series for the United States. *Journal of Geophysical Research*, **83**, 1377–1379.

HICKS S. D. and J. E. CROSBY (1974) Trends and variability of yearly mean sea level 1893–1972. National Oceanic Atmospheric Administration Ocean Survey, NOAA Tech. Memo., NOS 13, 14 pp.

HICKS S. D. and W. SHOFNOS (1965) Yearly sea level variations for the United States. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineering*, HY 5 (4468), 23–32.

LENNON G. W. (1975 to 1978) Monthly and annual mean heights of sea level. Permanent service for mean sea level, Institute of Oceanographic Sciences, Merseyside, 3 Vols (looseleaf).

LORENZ E. N. (1959) Empirical orthogonal functions and statistical weather prediction. Report 1, statistical forecasting project, Department of Meteorology, M.I.T., Boston, 49 pp.

MEADE R. H. and K. O. EMERY (1971) Sea level as affected by river runoff, eastern United States. *Science*, **173**, 425–428.

MERCER J. H. (1978) West Antarctic ice sheet and CO_2 greenhouse effect: A threat of disaster. *Nature, London*, **271**, 321–325.

MILLIMAN J. D. and K. O. EMERY (1968) Sea levels during the past 35,000 years. *Science*, **162**, 1121–1123.

PREISENDORFER R. W., F. W. ZWIERS and T. P. BARNETT (1981) Foundations of principal component selection rules. SIO reference series No. 81–4, Scripps Institution of Oceanography, University of California, La Jolla, 192 pp.

REDFIELD A. C. (1967) Postglacial change in sea level in the western Atlantic Ocean. *Science*, **157**, 687–692.

TARR R. S. and L. MARTIN (1912) The earthquake at Yakutat Bay, Alaska in September, 1899. U.S. Geological Survey Prof. Paper 69, 135 pp.

WILKINSON J. H. and C. REINSCH (1971) *Linear algebra*. Springer-Verlag, New York, 439 pp.

WINANT C. D., D. L. INMAN and C. E. NORDSTROM (1975) Description of seasonal beach changes using empirical eigenfunctions. *Journal of Geophysical Research*, **80**, 1979–1986.