# A CCD Chirp-Z FFT Doppler signal processor for laser velocimetry

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Abstract. A charge-coupled Chirp-Z fast Fourier transform device which has been configured to obtain the frequency of peak spectral amplitude in signals from laser velocimeters is described for use in oceanographic and other low laser power field applications where weak optical signals and frequent dropout are encountered. With sufficient bandwidth to resolve photon arrivals, and with a pulse-height discriminator, the processor is equivalent to a photon-correlator. The resulting signal processor gives 8-bit resolution in the velocity word, with a cycle time of approximately 4 ms while scanning a spectral range of 0-125 kHz; higher frequency ranges are possible at proportionately faster spectral scan rates, up to a maximum of 2 MHz. Error estimates are presented which indicate 2-bit accuracy at a signal-to-noise power ratio of -2 dB. Intercomparison data with discontinuous signals fed to this processor and a commercial frequency tracker are presented. The results demonstrate, conclusively, the superior accuracy of this processor. The combination of this processor together with quadrature demodulation methods described by the author (Agrawal 1984) offer an inexpensive directional LDV system with zero frequency offset.

#### 1. Background

In recent years, boundary layer research on the seafloor at depths of 5000 m has lead to the development of an autonomous, optically scanning backscatter laser Doppler velocimeter, which records the two horizontal velocity components at several heights off the seafloor up to 0.5 m (Agrawal and Terry 1982). During this development, one of the major difficulties in Doppler signal processing has been associated with the weak signals received from a 15 mW laser in an LDV operating at a range up to 1.0 m in backscatter mode, with the receiving aperture limited to a dimension of approximately 87 mm (this latter restriction is forced by the use of a schlieren window in the underwater housing at an external pressure of nearly 10 000 PSI (~69 MPa)). The system design incorporates a constant anode current photomultiplier (PMT) power supply, the purpose of which is dual: protection of the PMT from exposure to the sunlit deck of the ship, but more importantly, to act as an automatic gain control (AGC) to compensate for changes in water turbidity, or range of measurement. We found the phase lock loop frequency tracker (PLL) inappropriate owing to frequent signal drop-out, and experience with the counter-type processors was discouraging owing to the poor optical SNR as well as the use of the AGC in our system leading to false counts. An exhaustive search of the literature turned up no appropriate solution for the combination of frequent drop-out and poor SNR application together with compactness and low power consumption. It was determined that the most reliable methods of avoiding spurious data, when no knob-twiddling can be done following launch, would be to

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carry out the complete spectral decomposition of incoming signals, pick off the frequency of maximum spectral energy, and record this on tape. Doppler signal processors based on spectral analysis have been developed earlier. The obvious solution may appear to be a photon correlator, however, these devices are not compact and consume large amounts of power thanks to the fast electronics required. A burst processor such as one described by Brown et al (1979) is impractical for the application at hand. Similarly, a surface acoustic wave (sAw) processor described by Aldritt et al (1978) cannot be used for the small frequency range of use here because of the scaling of the acoustic sAw device. The solution we have chosen carries out the power spectral density (PSD) of the photocurrent computation purely in an analogue manner. Adrian (1980) has shown, and as is clear by definition, that if photoelectron pulse-height distribution is removed by a pulse-height discriminator, the analogue PSD is identical to the digital photon correlators. Thus, an analogue burst photon correlator is effectively possible, capable of the power of its digital counterparts, with considerably lower complexity, cost and power consumption. The processor described in the following is thus comprised of two major subsystems: a fast Fourier transform (FFT) unit and a peak determination unit. Data are recorded as 8-bit frequency words on tape, also a part of the autonomous seafloor instrument. Comparison of this device with a phase-lock device is presented.

#### 2. Description

The FFT is carried out in the commercially available EG & G Reticon RC5601 spectral density unit, which employs the Chirp-Z algorithm. Briefly, this algorithm involves expressing the Fourier transform as follows (Oppenheim and Schafer 1975):

$$F(\omega) = \int f(t) e^{j\omega t} dt$$
 (1)

where f(t) is the time series whose Fourier components are in frequency space  $\omega$ .

Equation (1) is expressed as a finite sum, at discrete values of frequency, i.e.  $\omega = n\Delta\omega$ ,  $t = m\Delta t$ , to give

$$F(n) = \sum_{m=0}^{N} f(m) \exp(jnmp)$$
(2)

where  $p = \Delta t \Delta \omega$ , and the identity

 $nm = \frac{n^2 + m^2 - (n - m)^2}{2}$ 

is employed to rewrite equation (2) as

$$F(n) = \sum_{m=0}^{N} f(m) \exp\left(j \frac{n^2 + m^2 - (n-m)^2}{2} p\right)$$

or

$$F(n) = \exp j\left(\frac{n^2 p}{2}\right) \sum_{m=0}^{N} \underbrace{f(m) \exp\left(j \frac{m^2}{2} p\right)}_{g_n} \exp\left(j \frac{-(m-n)^2}{2} p\right). (3)$$

Here N represents the total number of time samples or, identically, the number of spectral points and equals 512 for the RC5601. It is trivial to show that p = 1/N. The expression in the summation sign labelled  $g_n$  represents the multiplication of the incoming signal with a chirp signal  $\exp(jpm^2/2)$ . Convolution of  $g_n$  represented by the second multiplication is carried out by the EG & G Reticon R5601 Quad Transversal Filter. The real and imaginary parts of the sum of separately squared and summed to obtain the power spectral density  $F_0(n)$  (Reticon actually employ an approximation in the evaluation of  $F_0(n)$ ). The series  $F_0(n)$  is further approximated as

$$F_0(n) \simeq \sum_{m=n}^{N+n} g_n \exp\left(\frac{-(m-n)^2}{2}p\right).$$
 (4)

Thus each spectral point is successively updated with a new time sample included and the 'oldest' time sample discarded (in this regard, this processor differs from a photon correlator).

The quantity  $F_0(n)$  is output from the RC5601 as a sequence of 512 frequency points. This power spectral density series  $F_0(n)$ , is input into an electronic circuit to pick off the frequency of peak spectral amplitude. The operational block diagram (figure 1) is simple, and explained as follows: the incoming spectral amplitude  $F_0(n)$  is compared in a comparator with the last highest value, say  $F_0(a)$ . When  $F_0(n) > F_0(a)$ , the comparator output goes high, and the sample-hold chip acquires this value  $F_0(n)$  which in turn is used as the updated  $F_0(a)$  for the next incoming spectral amplitude. The comparator output pulse also causes the parallel-to-serial shift register to grab the digital 9-bit word which is used by the RC5601 to generate the frequency sweep. This 9-bit word is a digital representation of frequency. At the end of scan, the digital frequency word 'n' is latched and read into the buffer memory of a computer. In figure 2, a typical spectrum of the Doppler signal is shown, along with the output of the sample-hold circuitry  $F_0(a)$ .



**Figure 1.** Block diagram of the circuit for determining the frequency of peak spectral amplitude.

Some peculiarities of this system are worth noting. First, since most signals are real, corresponding to the existence of  $\pm \omega$ , the complete spectral density output includes the reflection of the 'desired' (real) spectrum at the highest scan frequency F(N). This can be eliminated by either (i) scanning only half the spectrum, thus ignoring the reflection, or (ii) by feeding the RC5601 the complex signal  $f_c(t)$ . In the latter case, electronic mixing of the Doppler signal with two signals out a quadrature must be employed to generate the real and imaginary components of  $f_c(t)$ . Agrawal and McCullough (1981) and Agrawal (1984) describe a photodiode array detector and other configurations which directly generate this complex signal. This arrangement allows use of the full frequency range sweep, increasing frequency resolution by a factor of two over the case of a real signal alone (the factor of two is gained on an expanded frequency scale).

As most signals in practice are real, only half the bandwidth swept is useful. For this case, the peak detection circuitry is disabled at n=256; an additional 'computer ready' gate may also be employed to hold data until it has been read into a computer memory. Second, by adding a DC offset  $[F_0(0)]$  to the PMT signal, the peak-detecting circuit will not update  $F_0(a)$ 



**Figure 2.** The Fourier spectrum of an incoming signal (lower trace), and the peak position detecting sample-hold output (upper trace). The full 512-point spectrum is displayed between arrows. The second strongest peak is the reflected frequency,  $\omega$ . In our set-up with real signals, only the first half spectrum is used. The small peak at left arrow is the DC spectral threshold. (The same threshold at the right arrow appears diminished due to aliasing of the digital oscilloscope used.)

unless a spectral value  $F_0(n) > F_0(0)$  occurs. Thus, a spectral threshold is arranged. This DC bias is set by low-pass filtering the spectrum itself.

The question of the accuracy of the peak frequency estimate obtained in this manner must be addressed. Typically, the Doppler signal spectrum is of Gaussian shape, whereas the background shot (and other) noise is white. Since one commonly observes the FFT spectrum on an oscilloscope, the following normalisation is used: the Doppler spectrum is normalised to its peak value, and the noise spectral density is also normalised to this peak (being flat, it is called  $\mathcal{N}$ ). Thus the Gaussian Doppler spectrum can be represented as

$$\exp[(\omega-\omega_0)^2T^2/2]$$

which implies an envelope of the signal in time which is also Gaussian, of FWHM of 4T. The total Doppler signal spectral energy is

$$\int_{-\infty}^{\infty} \exp(-(\omega-\omega_0)^2 T^2/2) \,\mathrm{d}\omega = \sqrt{2\pi}/T.$$

The noise spectral energy is simply the bandwidth B times the noise level  $\mathcal{N}$ . B may be made, at the narrowest, equal to the frequency scan range  $N\Delta f$ . This gives, using  $T = M\Delta t$ 

$$SNR = \sqrt{2\pi/TB}\mathcal{N}$$

$$= \sqrt{2\pi/M}\mathcal{N}.$$
(5)

Next, we compute the error bits estimated for the whitenoise level  $\mathcal{N}$ . The approach is simply to determine the  $\Delta \omega = \omega - \omega_0$  for which the Gaussian Doppler spectral amplitude drops by  $3\mathcal{N}$  where  $3\mathcal{N}$  is taken as the maximum noise contribution to instantaneous total spectrum (99% confidence

## Y C Agrawal

3

level). It is straightforward to show that the error bits will be

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$$e \leq \frac{512}{\sqrt{2\pi}M} \sqrt{\ln}[1/(1-3\mathcal{N})]. \tag{6}$$

As an example, let a scan bandwidth of 125 kHz be used, with 4T = 1 ms, or M = 31.2, i.e. a 1 ms long burst. The sNR, if  $\mathcal{N} = 0.1$  is equivalent from equation (5) to 0.8 or -2 dB. The error bits at such poor SNR will be, from equation (6),  $e \leq 2$ , which is really quite remarkable.

Finally, it is worth mentioning that there occurs a maximum uncertainty in knowing the exact time that a velocity realisation occurred by the amount of time required for one spectral scan, which is equal to N/B where B is the scanning bandwidth. In the example used above, B = 125 kHz which corresponds to an uncertainty of ~4 ms. In the case of continuous signals, or in order to reduce this uncertainty, a limited range of the spectrum may be computed by preloading the frequency-scan generating counters. The highest bandwidth which can be processed on the R5601 Quad Transversal Filter is 2 MHz with proportionately smaller processing time and uncertainty in time of velocity realisation. The total power consumption, of importance in field instrumentation or oceanography, is 12 W.

### 3. Test results

A two-axis system has been incorporated into the seafloor LDV. The two FFTs are operated synchronously, which implies that one of them is used as a master. Data are written on a Sea Data cassette recorder as velocity pairs (u, v). Each physical record on tape is written once per second, and consists of 15 (u, v) pairs, sampled every 30 ms (only the first velocity word output by the master FFT during the 30 ms window is seen, a coincident value if available, or a zero is written from the 'slave' or second axis). For the purpose of cross comparison, the slave FFT was replaced by a DISA model 55N20 Tracker. Identical signals were fed to the two units. Figure 3 shows a comparison of the output of the two devices, a swept frequency being the input. Signal pulses 6.5 ms long, repeated each 12.5 ms were generated

from an oscillator; the output frequency of the oscillator was swept from 10 kHz to 50 kHz in a 10 s period. The input signal and FFT scan were asynchronous. It can be seen that the FFT 'tracks' the velocity accurately, while a larger jitter appears on the tracker output, especially at the low end of the sweep. The FFT maintained the  $\pm 1$  bit accuracy with pulse lengths of only 0.25 ms, whereas the tracker predictably could not obtain lock. A subsequent series of tests were carried out, but with fixed frequency, with noise added and pulse width being the variable. In figure 4, where input signal to noise power was measured as 0 dB, the FFT continues to show  $\pm 1$  bit error whereas the tracker error increased to  $\pm 4$  bits (these data are for pulse lengths of 1 ms, repeated every 12.5 ms). We would have liked to make similar comparisons with a photon correlator, however, such a processor was not available.

The test data above definitively establish the accuracy of the FFT analyser-peak position detector. In view of the close match with the predictions of equation (6), it would be reasonable to expect that bandpass filtered analogue processing would perform as well as photon-correlation processors so long as prepulse height discrimination bandwidth is sufficient to distinguish photon arrivals.

#### 4. Summary

A Doppler signal processor for weak optical returns is described. The implementation involves the computation of the complete spectrum of the photocurrent, and picking the frequency of highest energy content. Error estimates show that at SNR below 0 dB, the processor works with remarkable accuracy. This device is most suitable for backscatter applications with low transmitted optical power, and with frequent signal drop-out, a condition in which no commercially available Doppler signal processors were found to perform adequately.

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460

## CCD Chirp-Z FFT Doppler signal processor for LDV

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3

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