Non-linear Tidal Distortion in Shallow Well-mixed Estuaries: a Synthesis^a

Carl T. Friedrichs and David G. Aubrey

Woods Hole Oceanographic Institution, Woods Hole, MA 02543, U.S.A.

Received 10 March 1987 and in revised form 11 May 1988

Keywords: tidal distortion; estuaries; overtides; tidal harmonics; sediment transport; lagoons

The importance of asymmetric tidal cycles in the transport and accumulation of sediment in shallow well-mixed estuaries is well established. Along the U.S. Atlantic Coast, tidal amplitude, bottom friction, and system geometry determine tidal distortion as documented at 54 tide gauges in 26 tidally dominated estuaries of varying geometry having negligible freshwater inflow. Analyses of sea-surface heights are compared to the results of one-dimensional numerical modelling to clarify the physics of tidal response in well-mixed estuaries. Concise measurements of estuarine geometry and ocean tidal range are used to predict consistently the nature of tidal sea-surface distortion. Numerical modelling then is utilized to extend theoretical and observational relationships between geometry and sea-height to predict trends in velocity distortion and near-bed sediment transport. Non-linear tidal distortion is a composite of two principal effects: (1) frictional interaction between the tide and channel bottoms (reflected in a/h =tidal amplitude/channel depth) causes relatively shorter floods; (2) intertidal storage (measured by V_s/V_c = volume of intertidal storage/volume of channels at mean sea level) causes relatively shorter ebbs. Variations in V_s/V_c and a/h trigger consistent and predictable changes in tidal distortion as measured through the first harmonic of the principal tidal constitutent.

Introduction

Estuarine coastlines (lagoons, bays, inlets, tidal flats, and marshes) make up 80–90% of the US. east and Gulf of Mexico coasts and occur on every continent. Accelerated development of these shallow water systems has highlighted concerns over eroding property, unstable shipping channels, and generally deteriorating environmental quality. Estuarine systems also play a significant role in the sediment cycle, exchanging material between land and sea and often acting as sediment sinks. The importance of asymmetric tidal cycles in the transport and accumulation of sediment in shallow estuaries is well established (e.g., Postma, 1967; Boothroyd & Hubbard, 1975). 'Flood-dominant' lagoons and estuaries (having shorter duration, higher velocity floods) tend to infill their channels with coarse sediment. 'Ebb-dominant' systems (having shorter, higher velocity ebbs) tend to flush bed-load sediment seaward more effectively and may represent more stable geometries

"Contribution number 6784 of the Woods Hole Oceanographic Institution.

0272-7714/88/050521+25 \$03.00/0

© 1988 Academic Press Limited

(Speer & Aubrey, 1985; Aubrey, 1986a). A wide range of both types of systems is found along the U.S. east coast. Although previous observational studies have been published on the effects of asymmetric tides on sediment transport, relatively few have investigated the estuarine properties that cause and control the degree of tidal distortion. To identify and quantify these estuarine relationships better, this study empirically relates measured seasurface tidal distortions to the geometries of a wide range of well-mixed, tidally dominated estuarine systems. Hypotheses to explain tidal asymmetries are discussed, and empirical observations are compared to predictions of one-dimensional numerical modelling.

The distortion of the tide as it propagates from the open ocean into the confinement of estuaries can be represented by the non-linear growth of compound constitutents and harmonics of the principal astronomical tidal components (e.g., Dronkers, 1964; Uncles, 1981; Speer & Aubrey, 1985; Boon in press). Since the ocean tide is a sum of sinusoidal components, non-linearities (such as a dependence on the square of the ocean tide) will produce harmonics and compound constituents. Transfer of energy to even harmonics can produce asymmetric tidal velocities and net transport of coarse bed-load sediment. Along much of the world's coastlines (including the east coast of the U.S.), the dominant astronomical constituent is M_2 , the semi-diurnal lunar tide. Because of M_2 dominance, the most significant overtide formed in these well-mixed estuaries is M_4 , the first harmonic of M_2 .

Within an estuary the distorted sea-surface height, A, and tidal velocity, V, can be modelled by a superposition of M_2 and M_4 :

$$\mathbf{A} = \mathbf{a}_{\mathbf{M}_{1}} \cos\left(\omega \mathbf{t} - \theta_{\mathbf{M}_{2}}\right) + \mathbf{a}_{\mathbf{M}_{2}} \cos\left(2\omega \mathbf{t} - \theta_{\mathbf{M}_{2}}\right) \tag{1}$$

$$V = v_{M_1} \cos(\omega t - \varphi_{M_1}) + v_{M_2} \cos(2\omega t - \varphi_{M_2})$$
(2)

where t is time, ω is tidal frequency, a is amplitude of tidal height, v is amplitude of tidal velocity, θ is phase of tidal height, and φ is phase of tidal velocity. The sea-surface phase of M_4 relative to M_2 is defined as

$$2M_2 - M_4 = 2\theta_{M_1} - \theta_{M_4} \tag{3}$$

;*

A direct measure of non-linear distortion, the M_4 to M_2 sea-surface amplitude ratio, can be defined as

$$M_4/M_2 = a_{M_4}/a_{M_2}$$
(4)

Likewise the non-linear parameters for tidal velocity are $2\varphi_{M2} - \varphi_{M4}$ and v_{M4}/v_{M2} . An undistorted tide has M_4/M_2 amplitude ratios of zero. A distorted, but symmetric tide has a relative $2M_2 - M_4$ velocity phase of $\pm 90^\circ$ and $M_4/M_2 > 0$. If M_4 is locked in a velocity phase of -90° to 90° relative to M_2 with $M_4/M_2 > 0$ [Figure 1(a)], the distored composite tide has a higher velocity flood and is defined as flood-dominant. Assuming a linear relationship, a flood-dominant system has a sea-surface phase of 0° -180° [Figure 1(b)]. If M_4 is locked in a velocity phase of 90° -270° and a surface phase of 180° -360°, the relationship is reversed, resulting in an ebb-dominant system. In either case, the larger the M_4/M_2 ratio, the more distorted the tide and the more strongly flood- or ebb-dominant the system becomes. These relationships are summarized in Figure 2.

To illustrate potential trends in sediment transport resulting from tidal asymmetry, a speculative ratio of flood-to-ebb near-bed transport can be related to velocity M_4/M_2 and $2M_2 - M_4$ (Figure 3). This is done by applying the Meyer-Peter-Müller (1948) empirical equations relating volume transport rate of sediment to shear stress and integrating predicted sediment transport rates over the flood and ebb portions of the tidal cycle (Aubrey,



Figure 1. Model of a flood-dominant (stronger flood flow) distorted tide: (a) M_4/M_2 velocity ratio=0.3, $2M_2 - M_4$ relative velocity phase=0°; (b) M_4/M_2 sea-surface amplitude ratio=0.3, $2M_2 - M_4$ relative surface phase=90°. The parameters M_4/M_2 and $2M_2 - M_4$ describe the non-linear distortion of these tides. ..., M_2 ; ---- $M_2 + M_4$; _____, M_4 .

1986*a*; Fry, 1987). Although the Meyer-Peter-Müller method has been shown to be useful in nearshore applications (Goud & Aubrey, 1985), no argument is made here for its indiscriminate, universal application. In this case predicted transport rate is proportional to V³, but any relationship in which sediment transport rate is geometrically proportional to velocity will produce similar trends. The Meyer-Peter-Müller equations in particular apply only to abiotic, geometrically smooth beds with grains of uniform size. The application here assumes zero critical shear stress required for initiation of motion. This assumption leads to an underestimate of the transport asymmetry, but is a useful indicator of sediment transport system response (particularly when shear stress >> critical shear stress over most of the tidal cycle). Results (Figure 3) indicate a net flood transport for relative phases within 90° of zero and net ebb transport elsewhere. Flood and ebb transport are equal only when relative velocity phase is at 90° or 270°. For magnitude of non-linearity observed in the field (up to velocity $M_4/M_2 \sim 0.25$), extreme flood-to-ebb transport ratios reach 2.25:1 at $2M_2 - M_4 = 0^\circ$ and 1:2.25 at $2M_2 - M_4 = 180^\circ$.

For an ideal investigation of tidal distortion directly relevant to sediment transport, field observations would include long-term records of tidal velocities in many shallow estuaries. However, existing tidal velocity sets consist largely of hourly measurements over cycles of only 12 or 24 hours. Samples of such short duration seriously limit the resolution of any analysis and cannot be averaged to represent an estuary under typical conditions. Where direct measurements of tidal velocities are too short or non-existent, observations of tidal height can provide a useful first approximation of net sedimentation patterns. As shown by Fry (1987), distortions in tidal height at the landward end of a tidal channel place continuity constraints on distortions in tidal velocity. Along the U.S. Atlantic Coast long-term sea-surface data have been collected at hundreds of permanent tide station (NOS, 1984), including many shallow estuaries. Thus by utilizing existing sea-surface data from several locations, sources of system geometry such as bathymetric maps, and one-dimensional numerical modelling, velocity trends in tidally-dominated estuaries can be predicted effectively.

Previous work on causes of tidal asymmetry

Hypotheses have been put forward to explain qualitatively flood- and ebb-dominant estuaries in isolation. In the absence of friction, flood dominance has been attributed to



Figure 2. Linear relationships between relative phase and tidal distortion for $M_4/M_2 > 0$.

distortion of a non-reflected progressive tidal wave (e.g., Saloman & Allen, 1983; Dronkers, 1986). In a frictionless estuary where a/h (tidal amplitude/water depth) is large, the tide may propagate as a shallow water wave, at a velocity $c = \sqrt{gh}$. As h varies over the tidal cycle, water depth is significantly greater at the tide crest than at the trough. Thus the wave crest tends to move more quickly than the trough through the length of a shallow estuary. The crest of the tide may partially overtake the trough, resulting in a shorter flood, a longer ebb, and the occurrence of highest velocity currents during the flood (due to conservation of mass). This effect was noted in the last century by Airy (1842). However, tidal propagation in many small estuaries (where length of estuary < < tidal wavelength)





is complicated by co-oscillation due to tidal wave reflection from the head of the embayment. To explain more fully flood dominance in shallow estuaries, friction must be considered.

In modelling tidal distortion due to frictional effects, stress on the sea bed is often expressed as a non-linear function (e.g. Uncles, 1981; Speer & Aubrey, 1985). Non-linear friction results in greater frictional damping in shallow water, slowing the propagation of water level changes around low tide relative to high tide (Dronkers, 1986). Thus the time delay between low water at the inlet and low water in the inner estuary is greater than the time delay between high water. The result inside the estuary is a longer ebb, a shorter flood, and highest velocity currents during the flood. As the a/h ratio or distance into such an estuary increases, this effect should increase.

Ebb dominance has been attributed to inefficient water exchange around high water in estuaries with relatively deep channels and extensive intertidal water storage (Nummedal & Humphries, 1978; Boon & Byrne, 1981; Speer & Aubrey, 1985). In estuaries where V_s/V_c (volume of intertidal storage/volume of channels) is large in relation to a/h, low velocities in intertidal marshes and flats cause high tide to propagate slower than low tide. At low tide marshes and flats are empty while channels are still relatively deep, allowing a faster exchange of water. The delay in the turn to ebb on the flats causes a relatively shorter ebb, longer flood, and highest velocity currents during the ebb. Boon and Byrne (1981) point out that ebb dominance may be enhanced further by variations in inlet cross-sectional area over the tidal cycle. If velocity near the inlet is in phase with discharge from the inner estuary, velocity may lag behind changing inlet cross-sectional area, which is in phase with the sea tide. In this situation, inlet area will always be smaller during ebb than during flood, forcing higher velocity ebbs.

A simple one-dimensional numerical model well suited for testing the relative importance of geometric and frictional influences on tidal distortion has been developed by Speer

and Aubrey (1985). Consistent with conceptual models, numerical results of Speer and Aubrey indicate that channels without intertidal storage in flats or marshes are flooddominant, while channels with intertidal storage great enough to overcome the effects of channel friction are ebb-dominant. However, the critical area of tidal flats needed to produce a longer rising tide was not parameterized because sparse field evidence could not provide sufficient guidance. In order to quantify the sensitivity of tidal distortion to real variations in forcing tides and system geometry better, this paper surveys existing field studies and utilizes previously unanalysed sea-surface data from diverse locations along the U.S. Atlantic Coast. Numerical models are run to elucidate the physics behind the different types of distortion observed in the field.

Model formulation

The equations governing incompressible fluid flow are conservation of mass (continuity) and conservation of momentum (Navier-Stokes):

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} + \frac{\partial \mathbf{w}}{\partial \mathbf{z}} = 0$$
(5)

$$\frac{D}{Dt}\vec{q}(u,v,w) = \frac{-1}{\rho}\vec{\nabla p} + \frac{1}{\rho}\vec{\nabla \tau} + \vec{g}$$
(6)

where $\vec{q}(u,v,w)$ is the velocity vector, p is pressure, $\vec{\tau}$ is the stress tensor, \vec{g} is the acceleration of gravity, ρ is the density of water, and t is time. To apply these equations to a shallow well-mixed estuary, integration over channel cross-sectional area replaces the x-component of velocity with volume flux. Equation (6) becomes a scalar equation with $\partial/\partial x$ replacing $\vec{\nabla}$; the total derivative, D/Dt generates local and advective terms; and hydrostatic pressure distribution, $p = \rho gz$, is assumed. A more detailed derivation is given by Speer (1984). The form of the equations used in the numerical model is:

$$\frac{\partial U}{\partial x} + b \frac{\partial \zeta}{\partial t} = 0$$
(7)

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\frac{U^2}{A} = -gA\frac{\partial\zeta}{\partial x} - \frac{\tau_b}{\rho}P$$
(8)

where $\zeta(\mathbf{x},t)$ is the sea-surface elevation, b is channel width, $U(\mathbf{x},t)$ is the cross-sectional flux, τ_b is the average shear stress on solid boundaries, P is the wetted channel perimeter, and A is the channel cross-sectional area. The problem is closed mathematically by formulating friction as:

$$\tau_{\rm b} = \rho f \left| \frac{\rm U}{\rm A} \right| \frac{\rm U}{\rm A} \tag{9}$$

where f is a dimensionless friction factor. Principal non-linear effects in these equations enter through tidal interactions with estuarine geometry in the continuity equation and through quadratic friction in the momentum equation. Advection of momentum has been identified also as a source of non-linear tidal distortion (e.g., Hamilton, 1978; Uncles, 1981). However, harmonic analysis of non-linear terms in the momentum equation indicates that the advective term is small compared to friction and is not an important driving mechanism for tidal distortion in this model (Speer & Aubrey, 1985).





Figure 4. Trapezoidal model channel geometry (after Speer & Aubrey, 1985). All momentum is transported in the trapezoidal channel, and sloping tidal flats act in a storage capacity only. $b_1 =$ width of channel at its base; $b_2 =$ width of channel at its top; $b_3 =$ width of tidal flat; $h_1 =$ depth of channel; $h_2 =$ mean depth of water column.

A trapezoidal geometry is used to represent the estuarine channel, where width increases with elevation above the bottom (Figure 4). The model approximates an ideal shallow estuary having two distinct elements: (1) a trapezoidal channel that transports all the momentum of the system, and (2) shallow, sloping tidal flats that act in a storage capacity only. By including the tidal flats in the continuity equation alone, the strong drop in velocity seen as water flows across tidal flats is approximated. The Speer–Aubrey model assumes a small horizontal aspect ratio (channel depth/width < <1), long narrow channels (width/length < <1) and little freshwater runoff. Two-dimensional modelling of shallow estuaries (e.g., Masch *et al.*, 1977) may predict sea-surface levels more precisely. However the Speer–Aubrey model produces computer model runs of more reasonable cost and duration. More importantly, because the inputs to the Speer–Aubrey model may be adjusted quickly, it can examine easily the roles of the parameters that actually control tidal distortion. Once preliminary quantitative relationships between tidal forcing, system geometry and tidal distortion are established, more complex modelling can be used eventually to refine the results.

Results

To measure tidal distortion in estuaries (Figure 5), sea-surface elevations were obtained over a full lunar cycle (697 hours) or more at all but one of 26 systems. Hourly sea heights were obtained from two sources: existing observational studies and the National Ocean Survey data banks. The method of least squares harmonic analysis was used to extract tidal components from the sea-surface data (Boon & Kiley, 1978). For the least squares method, no requirements are placed on the length of the sea-surface record nor on the sample interval (Speer & Aubrey, 1985). However, tidal records extending over one full lunar rotation are convenient for approximating stable values of constitutent amplitudes. For continuous series of 697 hourly observations, 29 tidal constitutents were extracted. For 73-hour series (beginning every 40 hours) which were utilized for the study of a/h variation over a single month, only four of the major components were extracted because of limitations in accuracy and resolution.

Geometric and tidal properties of the estuaries studied exhibit considerable variation (Table 1). The estuarine systems are quantified physically by length, offshore tidal amplitude, average depth and total volume of channels below mean sea level, volume of storage in intertidal flats and marshes, and position of tide gauge. Channels are defined as areas of the estuary submerged at mean low water. Intertidal storage volume may be determined in





several ways. If marshes play an insignificant role in storage and good topographic information is available, intertidal storage is approximated from tidal amplitude in the estuary and area of tidal flats. Otherwise intertidal storage is estimated from tidal prism minus the product of estuary tidal range and channel area at low tide. Tidal prism is obtained from existing direct measurements where possible or otherwise from the empirical equation of Jarrett (1976) relating tidal prism to inlet cross-sectional area. The non-dimensional ratio a/h (offshore M₂ amplitude/average channel depth at mean sea level) characterizes the expected frictional interaction between tide and channel by weighting the effect of tidal height relative to channel shallowness. The V_s/V_c ratio (intertidal storage in flats and marshes/volume of channels at mean sea level) reflects the potential effect of nonmomentum-carrying water in storage and parallels the two distinct elements of the Speer-Aubrey numerical model. The M₄/M₂ amplitude ratio and $2M_2 - M_4$ relative phase quantify the nature and degree of tidal distortion in each system (Aubrey & Speer, 1985; Speer & Aubrey, 1985).

Estimates of geometric parameters from existing bathymetric maps and empirical equations clearly produce results of limited accuracy. Furthermore, non-linear tidal distortion may vary over time scales greater than one lunar rotation due to seasonal sea-level fluctuations (Aubrey & Friedrichs, in press). Nonetheless by comparing a sufficient number of systems, important qualitative and first-order quantitative relationships may be found between tidal amplitude, estuarine geometry and non-linear generation of tidal harmonics.

To determine trends in tidal distortion predicted by the Speer-Aubrey model under conditions of varying channel depth and tidal flat extent, 84 differing model systems were analysed. All the model systems are 7 km long, a characteristic length of the shallow estuaries examined. To control further the number of variable parameters, channel cross-sectional shape is kept directly proportional in all the model systems: channel width (b_2) is 120 times low tide channel depth (h_1) , and $b_2/b_1 = 0.5$ (see Figure 4). Channel geometry

and the slope of the tidal flats do not vary along model length, and tidal flats in the model systems all just drain completely at low tide. In each case, off-shore forcing consists of M_2 alone, held constant at 0.75 m, and the friction factor is fixed at f = 0.01. Within the model systems, the non-linear distortion parameters (M_4/M_2 amplitude ratio and $2M_2 - M_4$ relative phase) are always measured 1.5, 3.3, 5.0 and 6.8 km into the tidal channel. Computer-generated contours of mean sea-surface M_4/M_2 and $2M_2 - M_4$ for these four stations are displayed in Figure 5.

Gauge position, estuary length and asymmetric ocean tide

Although this study focuses on a/h and V_s/V_c as parameters determining non-linear tidal distortion in shallow estuaries, there are other factors which affect the degree and nature of distortion: position of the gauge within the estuary, length of the estuary and asymmetries in tidal boundary conditions. If non-linear distortion is attributed to a relative delay in propagation of high or low water into the inner estuary, it follows that degree of seasurface distortion should increase with distance in from the inlet. This general trend is consistent with field observations and model results [Figure 6(a and c)]. Ten of the 12 systems with multiple gauges as well as both model channels exhibit increased distortion upstream. Also a decrease in sea-surface distortion is forced near the inlet if one assumes a sinusoidal ocean surface boundary condition. The most significant deviations from mean model predictions can be attributed to proximity of the tide station to the inlet. Strathmere, Townsend, and Breach tide gauges, which exhibit M_4/M_2 surface ratios 4.5 to 27 times smaller than mean model predictions, are all within 400 m of the inlet. Model results and field observations in multi-gauge systems indicate surface $2M_2 - M_4$ relative phase is much less variable within any single estuary [Figure 6 (b and d)]. Numerical modelling of tidal velocity suggests that cross-sectionally averaged velocity $2M_2 - M_4$ is fixed along channels and that velocity M_4/M_2 does not go to zero at the inlet.

Observations and numerical modelling indicate that channel length can modify the nature of non-linear distortion. Delaware Bay, which is 215 km long, exhibits an M_4/M_2 ratio 50 times greater than that predicted for a 7 km model channel with similar a/h and V_s/V_c . Numerical modelling of systems which differ only in length suggests that a longer channel causes increased surface M_4/M_2 and favours flood dominance (Figure 7). This trend is consistent with the mean surface $2M_2 - M_4$ relative phase of 91° in Delaware Bay. Observations of several systems (Table 1) indicate that shallower estuaries exhibit significant distortion at relatively shorter lengths. Swan Pond River and North Channel at Nauset suggest that for a/h > 0.5, 3-4 km of channel will produce significant distortion. Shark River with an M_4/M_2 of only 0.018 suggests that for $a/h \sim 0.3$, a mere 4.4 km may lessen potential distortion. Breach (5.2 km) and Duplin (13 km) both have $a/h \sim 0.2$, comparable V_s/V_c and similar tide gauge location. However the M_4/M_2 ratio in the longer Duplin system is over three times that at Breach.

Existing asymmetries in the ocean tide can alter the nature of tidal distortion within the estuary. The influence of distortions produced on the shelf may overshadow estuarine-produced harmonics, especially at locations having relatively little sea-surface distortion. For example, the apparent weakly flood-dominant sea-surface just inside the tidal inlet at Strathmere $(M_4/M_2=0.015, 2M_2-M_4=129^\circ)$ and Townsend $(M_4/M_2=0.003, 2M_2-M_4=129^\circ)$ could reflect existing ocean surface distortion such as that measured on the ocean near Murrells $(M_4/M_2=0.008, 2M_2-M_4=124^\circ)$. Ocean surface tidal distortion is increased further in sounds and straits. The anomalous $2M_2 - M_4$ phase just inside the inlet at Swan Pond River may reflect interaction with asymmetric tides in Nantucket Sound.

Estuary	I	Length of estuary (km)	Average channel depth (m)	Ocean M ₂ ampl. (m)	a/h	Channel volume (10 ⁶ m ³)	Flat and marsh storage (10 ⁶ m ³)	V _s /V _c	Position of gauge (km)	t _o , for single months; otherwise duration	M_4/M_2	$2M_2 - M_2$ (deg)	Data sources
1. North Chani	nel,	3.1	1.0	0.99	0.99	0.76	0.45	0.59	ocean	29/09/82-28/11/82	0.007	275	Aubrey & Speer, 1985".";
Nauset, MA									1.4	13/09/81	0.189	60	Aubrey & Speer, 1984 ⁶
									2.6	25/08/82;18/10/82	0.260	58	
2. South Chanr	nei,	0.8	2.1	0.99	0.41	0.4	1.5	0.19	ocean	29/09/82-28/11/82	0.007	275	Aubrey & Speer, 1985;**;
Nauser, MA									0.8	23/09/82;00/10/82	0.083	64	Aubrey & Speer, 1984
									1.8 3∙0	18/10/82 25/08/82;23/09/82; 18/10/82	0·119 0·142	64 64	
									4.6	17/03/83:26/06/83	0.142	59	
									6.0	22/07/82:17/03/83	0.207	63	
3. Chatham, M	A	14.0	2.4	0.99	0.41	44·0	13.0	0.30	ocean	29/09/82-28/11/82	0.007	275	Aubrey & Speer, 1985:4.
									5.1	15/05/87;21/01/88; 11/09/87	0.047	101	Aubrey unpublished data ^{<i>a</i>,<i>b</i>} ; NOAA chart 13248 ^{<i>a</i>}
									14.0	28/03/87	0.206	58	· · · · · · · · · · · · · · · · · · ·
 Swan Pond I MA 	R.,	4.5	0.8	0.42	0.53	0.53	0.06	0.11	0.1	30/06/87-14/07/87	0.133	-11	Aubrey, 1987 ^{<i>a,b</i>}
									4·2	30/06/87-14/07/87	0.087	91	
 Stony Brook NY 	,	5.2	1.7	0.86	0.51	4 ·8	1.3	0·27	ocean	05/11/83	0.035	57	Park, 1985";
									1.0	09/09/83;05/11/83	0.020	41	Zarillo & Park, 1987;
									2.1	09/09/83;05/11/83	0.128	45	USGS topo St. James, NY ^b ;
									4 · 1	09/09/83	0.200	56	NOAA chart 12364
6. Shark River,	,NJ	4.4	1.9	0∙60	0.32	4·5	0.49	0.11	2.8	01/01/77	0.018	126	NOS, 1984°; USGS topo Asbury Park, NJ ^b
7. Manasquan,	NJ	9·2	1.5	0∙58	0.39	5.9	0.57	0.10	ocean	01/09/82	0 012	186	NOS, 1984"; USGS topo
						·			6 ∙0	01/05/76	0.099	50	Pt. Pleasant, NJ ^b
8. Absecon, NJ	ſ	8∙0	2.9	0.26	0.19	29.0	23.0	0.79	2.2	01/06/78	0.028	262	NOS, 1984 ^a ; Jarrett, 1976 ^b ; USGS topos Oceanville/
9. Strathmere,	NJ	5.3	2.3	0.56	0.24	5.4	5-1	0.94	0.5	01/06/78	0.015	129	NOS, 1984"; Jarrett, 1976 ^k ; USGS topos Sea Isle City, NI
10. Townsend, 1	NJ	5.8	2.3	0.57	0.25	7 ∙3	8.3	1.14	0.4	01/06/78	0.003	135	NOS, 1984 ^e ; USGS topos Sea Isle City/Avalon, NI ^b
11. Delaware Ba	y	215	5.7	0.65	0.11	12 000	n.a	~0	ocean	1977-1983	0.017	280	Parker, $1984^{a,b}$
									45		0.041	72	
									85		0.074	98	
									130		0.125	93	
									155		0.094	113	
									215		0.142	81	
12. Northam, VA	A	4.7	1.6	0.50	0.31	1.3	1.1	0.85	3.1	01/09/82	0.065	, 143	NQS, 1984 ^a ; USGS topo Bloxom, VA ^b

.

TABLE 1. Estuary properties

					• •		· · · · · · · · · · · · · · · · · · ·	بحثاريهما ويهولا أرار		المترجودي والمستشرعين والمعالية والمعالمة والمستور والمستقد والمستقد	ويعطف والمراسية والمراجع	فللمعمدة المعدمان	، ۲۰۰۰ ، ۴ ۵۰٬۰۰۰ ۲۵٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰٬۰۰۰ ۲۰
3.	Wachapreague,	13.0	4·2	0.54	0.13	51·0	66.0	1.29	9·5	01/01/83–31/12/83	0.042	203	Boon, pers. com.";
	VA												Byrne et al., 1975 ^b ;
													DeAlteris & Byrne, 1975
4.	Rudee, VA	1.1	4.5	0.48	0.11	1.0	0.13	0.13	0.4	01/04/84	0.011	193	NOS, 1984°; NOAA
					Í								chart 12205 [*]
5.	Little River, SC	13.0	2.6	0.64	0.25	9.9	7.2	0.73	51	01/03/75	0.037	77	NOS, 1984";
									8∙6	01/12/75	0.051	87	NOAA chart 11534°
5.	Main Creek,	8.0	1.9	0.73	0.38	3.0	3.4	1.13	ocean	01/10/74;16/12/74;	0.008	124	NOS, 1984";
	Murrels, SC									14/02/75-31/12/75	0.000		Aubrey & Friedrichs, in press;
									1.2	01/10/74-31/12/75	0.092	83	Perry et al., 1978
									4.7	15/10/74-31/11/74	0.070	98	
						0 50			8.0	01/10/74 - 31/01/74	0.101	90	NOC 1004:
'÷	Oaks Creek,	4.7	14	0.73	0.52	0.73	1.3	1.78	ocean	01/10/74-10/12/74;	0.008	124	NUS, 1984";
	Murrells, SC								0.0	14/02/75-51/12/75	0.077		Aubrey & Friedrichs, in press;
									0.8	01/10/74 - 31/01/76	0.001	11	Ferry et al., 1978"
									2.9	01/10/14-31/12/14;	0.041	97	
									1.7	01/02/75-50/04/75	0.092	112	
,	Month Inlat 60	6.5	2.4	0.72	0.30	0.9	0.0	1.01	4.1	01/10/74 - 51/01/70	0.0082	115	NOS 10949
5.	North Inlet, SC	0.5	2.4	0.75	0.50	9.0	9.9	1.01	ocean	14/02/75 21/12/75	0.009	124	NUS, 1964'; Nummadat & Humphrice
										14/02/13-31/12/13			1078 ^{<i>a,b</i>}
									0.6	01/04/76	0.043	207	NOA A chart 11535 ^b
									1.0	01/01/76	0.042	108	Norm chart 11555
									3.5	01/12/76	0.058	202	
,	Price SC	7.1	3.3	0.69	0.21	8.9	9.6	1.08	1.1	01/12/77	0.042	241	NOS 1984 ^a : Fitzgerald &
•	11100,000		55	0.07	021	0,	, ,		••	01/12/11	0012	211	Nummedal 1983 ^b : USGS
													topos
													Sewee Bay/Caper, SC
).	Capers, SC	6.9	3.3	0.71	0.22	13.0	8.8	0.68	1.1	01/01/76	0.036	226	NOS, 1984 ^e ; USGS topo
	· F · · · · · · · · ·									, ,			Sewee Bay, SC ⁺
1.	Dewees, SC	7.6	4·3	0.72	0.17	20.0	14.0	0.70	1.4	01/03/76	0.024	235	NOS, 1984"; USGS topos
	•												Capers/Ft. Moultrie, SC ^b
2.	Breach, SC	5.2	3.3	0.73	0.22	9.5	14.0	1.47	0.5	01/06/75	0.024	200	NOS, 1984 USGS topo
	-												Ft. Moultrie, SC ^b
3.	Folly, SC	11.0	3.6	0.76	0.21	17.0	15.0	0.88	4.2	01/08/77	0.057	209	NOS, 1984"; USGS topo
													James Island, SC ^b
1 .	Duplin, GA	13.0	4 ·7	0.99	0.21	7.9	7.2	0.91	0.6	not available	0.071	277	Zarillo, 1987 ^{a,b}
5.	St. Marys	18.0	5.7	0.90	0.16	280 .0	52·0	0.19	ocean	18/06/85	0.021	1	Aubrey, 1986b ^a ; NOAA chart
	Entrance, GA								4.1	18/06/85	0.040	318	11503 ^b
									8∙5	19/06/85	0.027	240	
									12.	19/06/85	0.028	252	
									17.	18/06/85	0.024	231	
	E. C EI	8.0	2.6	0.72	0.28	10.0	3.0	0.30	4 ∙0	01/05/81	0.032	147	NOS, 1984 ^e ; Jarrett, 1976 ^b ;
5.	Ft. George, FL	00		• • •									

^eSea heights. ^eGeometry.

531

Non-linear tidal distortion



C. T. Friedrichs & D. G. Aubrey

532

Figure 6. Parameters which determine non-linear tidal distortion as a function of relative position along estuarine channel: field observations of (a) surface M_4/M_2 amplitude ratio, (b) surface $2M_2 - M_4$ relative phase; results along two model channels for (c) surface and velocity M_4/M_2 amplitude ratio; (d) surface and velocity $2M_2 - M_4$ relative phase. Results indicate that M_4/M_2 is more strongly a function of guage position than $2M_2 - M_4$. 'A' and 'C' indicate geometry in Figure 5. (a,b) \Box , 1; \bigcirc , 2; \triangle , 3; \times , 4; \diamond , 5; \bigtriangledown , 7; \bigoplus , 11; \bigoplus , 16; \coprod , 17; \boxplus , 18; \blacksquare , 25; +, single guage. (c,d) - ---, A surface; ----, A velocity; -----, C surface; --+-, C velocity.

Observations from estuaries with multiple gauges [see Figure 6(a and b)] and modelling results (Figure 8) suggest that non-linear distortion within the estuary is controlled more strongly by estuary geometry than by asymmetries in the ocean surface boundary condition.

The remainder of this study focuses on a/h and V_s/V_c as parameters determining tidal distortion in shallow tidal estuaries. In investigating the role of a/h and V_s/V_c in generating tidal harmonics, the characteristic sea-surface distortion response of entire estuaries must be determined. For this reason, records extending over only a few tidal cycles were not considered. However, records of a month or more at a single location may still not reflect the characteristic response of the entire estuary. This is the case at three systems in Table 1 which combine shorter-than-average channel length with lone gauge positioning just



Figure 7. Parameters which determine non-linear tidal distortion as a function of channel length: model results for mean (a) surface and velocity M_4/M_2 amplitude ratio; (b) surface and velocity M_2/M_4 relative phase. Results suggest a longer system increases surface M_4/M_2 and favours flood dominance. 'B' indicates geometry in Figure 5. _____, B surface; ----, B velocity.



0

ıd

g

d

y

al g

st

١t

:t 1

st

Figure 8. Parameters which determine non-linear tidal distortion as a function of relative position along an estuarine channel for two model channels each subject to three ocean tides differing only in surface M_4/M_2 and $2M_2 - M_4$: (a) surface M_4/M_2 amplitude ratio; (b) surface $2M_2 - M_4$ relative phase. Characteristic estuarine distortion patterns are relatively unaffected, especially in the inner estuary. 'A' and 'C' indicate geometry in Figure 5. Ocean distortion: ---, $M_4/M_2=0$; ---, $M_4/M_2=0.05$, $2M_2 - M_4=90$; \cdots , $M_4/M_2=0.05$, $2M_2 - M_4=270$.

inside the inlet: Strathmere, Townsend and Breach. Because gauges at these three estuaries do not reflect significantly characteristic inner estuary sea-surface distortion, these estuaries are not considered further in analysing the effects of a/h and V_s/V_c . For shallow systems having several gauges, characteristic response is approximated better by averaging distortion parameters for individual tide stations. Also, the investigation concentrates on tidal estuaries of length common to barrier-beach coastlines. Since Delaware Bay

	Offset(β_0)	a/h Coef.(β_1)	V_s/V_c Coef.(β_2)
$M_{\rm A}/M_{\rm A}$ ratio (Y)	······································		
All systems	0·0055	0·24	0·0038
	[·67]	[>·9999]	[·64]
Flood-dominant	-0.010	0·26	-0·015
	[.67]	[·9999]	[·80]
Ebb-dominant	0.0008	0·13	0·019
	[.51]	[·86]	[·85]
$2M_2 - M_1$ phase (Y)	L J	(J	()
All systems	220	290	41
	[>-9999]	[>∙9999]	[·93]
Flood-dominant	120	-100	21
	[·9993]	[·96]	[·85]
Ebb-dominant	290 [·9996]		-41 [·88]

vs/vc

TABLE 2. Results of *t*-test relating estuary properties to parameters affecting non-linear tidal distortion^a

"The equations are in the form $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$, where $X_1 = a/h$ and $X_2 = V_s/V_c$. [] = probability that statistically 'true' coefficient is of same sign as β_0

(215 km) is highly anamolous, Delaware Bay is not included in further discussion of a/h and V_s/V_c .

Relative channel depth and intertidal storage

Multiple linear regression is used to relate general physical properties of estuaries that potentially may account for differences in tidal distortion (namely relative channel depth and intertidal storage volume) to the parameters that quantify non-linear tidal distortion $(M_4/M_2 \text{ amplitude ratio and } 2M_2 - M_4 \text{ relative phase})$. Utilizing data from Table 1, relationships of M_4/M_2 and $2M_2 - M_4$ phase vs. a/h and V_s/V_c were derived (Table 2) for all estuaries, all flood-dominant systems and all ebb-dominant systems. Because the seasurface is virtually undistorted at Rudee $(M_4/M_2 = 0.011, 2M_2 - M_4 = 193^\circ)$, this system is not labelled as either flood- or ebb-dominant and is included only in regressions of all systems. The probability that the best-fit coefficient (β) is of the same sign as the statistically 'true' coefficient is determined using the *t*-test. Numerical model results for sea surface based upon the one-dimensional equations of motion are consistent with empirical findings (Figure 9). The nature of tidal distortion in well-mixed estuaries is a compromise between the influences of frictional distortion in channels and intertidal storage in tidal flats and marshes.

Sea-surface M_4/M_2 [Table 2; Figure 9(a)]. The M_4/M_2 amplitude ratio is a direct measure of non-linear tidal distortion. According to both model and field observations of flood dominant systems, surface M_4/M_2 rises with greater a/h (as the relative difference in water depth at low and high water is increased). According to model results for flooddominant estuaries, as V_s/V_c increases from zero, tidal asymmetry is reduced initially as the non-linear effects of channel friction are compensated by additional water storage in tidal flats. Model results suggest that the non-linear distortion created by increased intertidal storage is not always opposite to the non-linear effects of channel friction. At sufficiently large V_s/V_c , M_4/M_2 no longer declines and eventually grows with further



Figure 9. Mean values for parameters which determine non-linear tidal distortion in 22 existing shallow estuaries (superimposed on results of numerical modelling) as a function of a/h and V_s/V_c : (a) surface M_4/M_2 amplitude ratio; (b) surface $2M_2 - M_4$ relative phase. Considering simplicity of modelling and geometric approximations, field observations agree well with model predictions.

addition of tidal flats. Field observations also suggest that the relationship between M_4/M_2 and V_s/V_c may be positive or negative. At large a/h increased V_s/V_c seems to decrease M_4/M_2 (e.g., Middle Channel at Nauset *vs.* Oaks Creek, Murrells), while at lower a/h increased V_s/V_c seems to increase M_4/M_2 (e.g., Ft. George *vs.* Northam Narrows).

In both model and field observations of ebb-dominant systems M_4/M_2 rises as V_s/V_c is increased. Absolute non-linear distortion is increased as additional intertidal storage enhances the existing asymmetry. Modelling of ebb-dominant estuaries suggests a positive relationship between a/h and M_4/M_2 at low a/h. Increasing a/h from low values may enhance ebb-dominant surface distortion because larger tidal amplitude increases the surface gradient between high water (delayed in the inner estuary by intertidal storage) and quickly propagating low water. However, as a/h increases with V_s/V_c held constant, M_4/M_2 eventually peaks and then decreases until it reaches a minimum coincident with the transition to flood dominance (i.e., the $180^\circ 2M_2 - M_4$ contour). At moderate a/h the delay of low water due to increased channel friction reduces ebb-dominant distortion as indicated by lower M_4/M_2 . Trends in field observations agree with model results at low a/h. Multiple linear regression of field data suggest a positive relationship between a/h and M_4/M_2 among ebb-dominant systems. The peak and decrease in M_4/M_2 at larger a/h is not seen, perhaps due to a lack of sufficient field data.

Examination of time series of the non-linear forcing terms in the numerical model confirms that in flood-dominant systems M_4/M_2 is due primarily to frictional distortion in channels (via the friction term); in ebb-dominant systems M_4/M_2 is more a result of intertidal storage (via the continuity term). The form of the friction and continuity terms (after Speer, 1984) are $f|\overline{u}|\overline{u} \cdot (P/A)$ and $1/b \partial/\partial x (A \cdot \overline{u})$, where \overline{u} is cross-sectionally averaged velocity. M_4/M_2 is determined for the non-linear forcing terms by harmonic analysis of the individual terms over a tidal cycle. Figure 10 illustrates variation in friction and continuity



Figure 10. M_4/M_2 ratios from harmonic analysis of time series of the non-linear forcing terms in the numerical model as a function of distance from inlet along estuary channel in (a) a flood- and (b) an ebb-dominant system. Results indicate that non-linearity is greater in the friction (continuity) term in the flood- (ebb-) dominant system. 'A' and 'C' indicate geometry in Figure 5. — + —, C friction; --+--, C continuity; — O^{\perp} , A friction; --O--, A continuity.

 M_4/M_2 along channel for a strongly flood-dominant and a strongly ebb-dominant system. Non-linearity in the friction term is greater in the flood-dominant system, reflecting greater frictional damping in shallow water and slower propagation of low water through the inner estuary. Non-linearity in the continuity term is greater in the ebb-dominant system, reflecting storage of water in intertidal flats and marshes and slower propagation of high water.

Sea-surface $2M_2 - M_4$ [Table 2; Figure 9(b)]. The $2M_2 - M_4$ relative sea-surface phase determines the orientation of tidal distortion (where $2M_2 - M_4 < 180^\circ$ indicates flood dominance and $2M_2 - M_4 > 180^\circ$ indicates ebb dominance). Consistent with field observations of ebb-dominant estuaries, model results suggest that increased V_s/V_c results in a rapid initial rise in $2M_2 - M_4$ from below 180° to above 280°, followed by a gradual decrease in $2M_2 - M_4$ with continued addition of intertidal storage. As absolute surface distortion (indicated by M_4/M_2) increases, $2M_2 - M_4$ does not approach a limit that would provide maximum asymmetry. As V_s/V_c rises in ebb-dominant estuaries, mean $2M_2 - M_4$ continues to fall to as low as 202°, even though (with M_4/M_2 held constant) greatest asymmetry in tidal sea-surface occurs at a relative phase of 270°. This is likely because the time required to fill and empty extensive tidal flats and marshes results in a long period of nearly stationary sea height symmetric about estuarine high water. Such a symmetric distortion is represented by $2M_2 - M_4 = 180^\circ$ and $M_4/M_2 > 0$. Thus increased intertidal storage may result in a movement of relative phase toward 180°. One significant exception to decreased $2M_2 - M_4$ with increased V_s/V_c is Duplin, where the tide station is near the mouth of the estuary and may be affected by existing distortion in neighbouring Doboy

Sound (Zarillo, 1985). Model results of ebb-dominant estuaries indicate relative phase falls as a/h increases. This reflects an increasing influence of channel friction on tidal distortion. Field observations also indicate sea-surface $2M_2 - M_4$ eventually falls drastically as a/h increases (i.e., the eventual transition to flood dominance). However among ebb-dominant estuaries the correlation between a/h and $2M_2 - M_4$ is only weakly negative.

Both the model results and multiple linear regression of field observations of flooddominant estuaries indicate sea-surface $2M_2 - M_4$ falls steadily as a/h increases or V_2/V_c decreases. In flood-dominant systems relative phase does not approach the limit that would provide maximum asymmetry either. As a/h rises or V_s/V_c falls, mean $2M_2 - M_4$ continues to fall to as low as 40°, even though greatest asymmetry occurs at 90°. Thus greater absolute tidal distortion in a flood-dominant estuary is reflected by a general decrease in $2M_2 - M_4$ relative phase away from 180°, rather than a movement in $2M_2 - M_4$ towards 90°. The probable reason for this behaviour is similar to the movement toward 180° in ebb-dominant systems. As friction grows at low water relative to high water, a longer period of nearly stationary sea height occurs symmetric about estuarine low water. Such a symmetric distortion is represented by $2M_2 - M_4 = 0°$ and $M_4/M_2 > 0$. Thus increased low water friction may result in a movement of relative phase toward 0°. Little River has a $2M_2 - M_4$ relative phase (82°) much lower than predicted for an estuary with a/h = 0.26. This result may reflect the greater than average length of the Little River system (13 km plus effects of the connected intracoastal waterway).

Spring/neap response. From the examination of estuaries having different system geometries some trends are clear, such as high $2M_2 - M_4$ at low a/h or high M_4/M_2 at high a/h. Other trends, such as the inverse relationship between relative phase and V_s/V_c among ebb-dominant systems appear less secure observationally. By utilizing the variation in surface $2M_2 - M_4$ and M_4/M_2 over the spring-neap cycle, these relationships can be examined more closely. Furthermore, studying changes in distortion at individual estuaries reduces such influences as varying channel length and gauge position among different estuaries. During spring tides, the increased tidal amplitude due to the constructive phasing of S_2 and M_2 enhances finite amplitude effects (a/h) on the propagating tide. However, intertidal storage (V_s/V_c) is increased simultaneously.

To illustrate possible pathways for estuaries to follow over the spring-neap cycle, a simple relationship is assumed between a/h and V_s/V_c , with both values going to zero at zero amplitude linearly (Figure 11). Linear regression relates a/h to surface M_4/M_2 and $2M_2 - M_4$ (Table 3) at tide gauges in those estuaries for which hourly sea heights were available. In each case 16 73-hour series (beginning every 40 hours) were utilized for the study of tidal variation over a single month. Variation in simultaneous ocean M_2 amplitude was used for calculating a/h where available (i.e. Murrells); otherwise, to approximate changes in a/h, variations in estuarine M_2 were scaled to the ocean tidal range. Because of limitations in resolution, harmonic analysis extracted only four of the major components (K_1, M_2, M_4, M_6). Thus M_2 and M_4 represent all semi-diurnal and quarter-diurnal sea-surface variance respectively.

Virtually all tide gauges indicate a positive correlation between M_4/M_2 and a/h [Table 3, Figure 12(a)]. Among flood-dominant systems this correlation reflects increased frictional interaction with channel bottom at high a/h. Among ebb-dominant systems with large V_s/V_c , $a/h-V_s/V_c$ paths [Figure 11(a)] suggest the positive relationship is due primarily to increased intertidal storage. Model results suggest a weak negative relationship only





Figure 11. Paths for a/h and V_s/V_c over the spring-neap cycle (superimposed on results of numerical modelling) assuming a simple relationship between a/h and V_s/V_c , with values of both parameters going to zero simultaneously and linearly. Numbers identify a/h and V_s/V_c values of estuarine systems in Tables 1 and 3. Surface (a) M_4/M_2 increases and (b) $2M_2 - M_4$ decreases along most a/h-V_s/V_c spring-neap paths.

T 1 1	M₄/M₂ equation		$2M_2 - M_4$ equation			
1 idal gauge coef.(β_1)	offset(β_0)	$a/h \operatorname{coef.}(\beta_1)$	offset(β_0)	a/h		
3. Chatham, MA #3	-0.016[.56]	0.98[.999]	66[>.9999]	- 15[·84]		
6. Shark River, NJ	0.035[.93]	-0.0091[.55]	110[-85]	97[-62]		
7. Manasquan, NJ	-0.021[.66]	0.34[.99]	57[-996]	- 11[.59]		
12. Northam Narrows,						
VA	0.0026[.53]	0.22[.97]	200[-9997]	- 170[·87]		
13. Wachapreague, VA	0.012[.73]	0.27[.97]	430[>.9999]	1500[-995]		
15. Little River, SC #1	-0.056[.93]	0.38[.99]	450[>·9999]	- 1200[-9993]		
#2	-0.016[.61]	0.33[.91]	150[.98]	-200[.77]		
16. Main Creek #2	-0.20[.9994]	0.91[>.9999]	33[-97]	100[.73]		
Murrells, SC #4	-0.16[.9998]	0.81[>.9999]	170[> 9999]	- 200[.99]		
17. Oaks Creek, #2	-0.16[.99]	0.54[.9996]	220[.93]	-310[·81]		
Murrells, SC #4	-0.086[.997]	0.42[>.9999]	130[>.9999]	-43[.78]		
19. Price, SC	0.0036[.60]	0.19[.99]	300[>.9999]	- 230[-94]		
20. Capers, SC	-0.017[.87]	0.25[.999]	180[-9999]	210[.88]		
21. Dewees, SC	0.034[.98]	-0.0042[.52]	390[> 9999]	- 760[-998]		
25. Ft. George, FL	-0.038[.98]	0.26[.9998]	260[-99]	350[.84]		

TABLE 3. Results of *t*-test relating a/h (M_2 tidal amplitude/mean channel depth) to parameters affecting non-linear tidal distortion⁴

"The equations are in the form $Y = \beta_0 + \beta_1(a/h)$. [] = probability that statistically 'true' coefficient is the same sign as β_0 .

among ebb-dominant systems having low V_s/V_c , which may account for the poor correlation at Capers. Generally negative offsets are consistent with model results also. Both observations and numerical modelling indicate surface distortion (as measured by M_4/M_2)



Figure 12. Distortion parameters as a function of a/h ratio over 16 73-hour cycles over the spring-neap cycle at Wachapreague (May 1983), Little River gauge \$1 (March 1975) and Main Creek gauge \$3, Murrells (May 1975): (a) surface M_4/M_2 amplitude ratio, (b) surface $2M_2 - M_4$ relative phase. +, Wachapreague, VA; ×, Little River 1, SC; \diamond , Main Creek 3, Murrells, SC.

is negligible for the initial segment of any $a/h-V_s/V_c$ path (below $M_4/M_2 \sim 0.01$) and then increases more or less linearly.

Most tide gauges indicate a negative relationship between $2M_2 - M_4$ and a/h along the a/h-V_s/V_c path [Table 3, Figure 12(b)]. Among flood-dominant systems at low V_s/V_c or ebb-dominant systems at high V_s/V_c, the decrease in $2M_2 - M_4$ reflects a longer period of nearly stationary sea height about low or high water respectively [Figure 11(b)]. For flood-dominant systems with high V_s/V_c model results suggest that effects of increased channel friction and increased intertidal storage may balance each other, resulting in indeterminant behaviour of relative phase. This may be the case at Main Creek gauge \$1, Murrells. Only for ebb-dominant systems at low V_s/V_c path. This may be the case at Dewees. However it is also possible that low $2M_2 - M_4$ at Dewees during neap reflects the influence of offshore $2M_2 - M_4$ in the absence of significant estuarine generated distortion.

Velocity distortion and sediment transport. Asymmetries in tidal velocity rather than seasurface control directly net sediment transport patterns. However existing observations of tidal velocity are too sparse to describe adequately the role of system geometry in determining the nature of distortion. Numerical modelling provides the ability to predict trends in non-linear tidal velocity in shallow estuaries based upon extensive field observations of tidal sea surface. From predicted distortions in tidal velocities, ratios of floodto-ebb near-bed transport ratios may be estimated. Figure 13(a and b) displays mean non-linear distortion parameters for cross-sectionally averaged tidal velocity as functions of a/h and V_s/V_c . As done for sea surface, velocity M_4/M_2 and $2M_2 - M_4$ are measured 1.5, 3.3 and 5.0 and 6.7 km into the tidal channel. The general features of distortion in tidal velocity follow trends similar to sea surface. The relative phase contour separating flood from ebb-dominant systems (270° instead of 180°) follows virtually the same path. As with sea-surface distortion, the transition from flood to ebb dominance follows a trough in M_4/M_2 . Among flood-dominant systems velocity M_4/M_2 increases with a/h as increased frictional interaction with channel bottom slows propagation of low water, and M_4/M_2 decreases initially with increased V_s/V_c as compensating intertidal storage delays high water.



Figure 13. Contour plots of the numerical results of 84 model systems as a function of a/h and V_y/V_c : (a) cross-sectionally averaged velocity M_y/M_2 amplitude ratio; (b) cross-sectionally averaged velocity $2M_2 - M_4$ relative phase; (c) flood-to-ebb near-bed transport ratio. Agreement between observations and modelling of sea surface distortion is utilized in predicting trends in velocity distortion and near-bed sediment transport.

Details of non-linear distortion in tidal velocity differ significantly from sea-surface distortion. Velocity M_4/M_2 is consistently higher than sea-surface M_4/M_2 . Linearized analytic solutions for model channels without tidal flats predict velocity M_2/M_4 to be two times sea-surface M_4/M_2 in the inner estuary (Fry, 1987). This approximation holds well for numerical solutions of most flood-dominant channels but not at all for ebb-dominant channels. According to numerical modelling of ebb-dominant systems, as intertidal storage increases from low value, velocity M_4/M_2 increases quickly (to an order of magnitude greater than surface M_4/M_2), reaches a broad maximum and decreases slightly at high V_s/V_c ; at moderate V_s/V_c velocity M_2/M_4 is solely dependent upon a/h and falls with decreased depth. Among ebb-dominant estuaries, numerical modelling suggests

increased V_s/V_c causes velocity $2M_2 - M_4$ to decrease away from a symmetric 270° and toward maximum ebb dominance in velocity which occurs at $2M_2 - M_4 = 180^\circ$. For flood-dominant systems, numerical modelling also predicts velocity phase moves close to the value of greatest distortion. Most flood-dominant systems display a velocity $2M_2 - M_4$ within 20° of a perfectly asymmetric 360°.

Figure 13(c) illustrates possible trends in sediment transport, given the velocity conditions of Figure 13(a and b). Flood-to-ebb near-bed transport ratios are based on the Meyer-Peter-Müller (1948) sediment transport equations, integrated over the tidal cycle. Although the Meyer-Peter-Müller method (transport proportional to V³) has been shown to be useful in nearshore applications (Goud & Aubrey, 1985), any relationship in which sediment transport rate is geometrically proportional to velocity will produce a roughly similar pattern. Results in Figure 13(c) assume zero critical shear stress required for initiation of motion and may underestimate maximum transport asymmetries in these systems. For example, if critical shear stress in a flood-dominant channel were greater than zero and were reached only during peak flood, then the flood-to-ebb bedload transport ratio would approach infinity. Figure 13(c) emphasizes the interaction of velocity $M_{a}/$ M_2 and $2M_2 - M_4$ in determining possible sediment transport. Among flood-dominant systems relative phase remains close to a strongly asymmetric 360°, thus the flood-to-ebb transport ratio reflects variations in velocity M_4/M_2 directly. However, among ebbdominant systems $2M_2 - M_4$ moves progressively away from a symmetric 270° as V₄/V_c increases. Thus the flood-to-ebb transport ratio more closely reflects variations in velocity $2M_2 - M_4$.

Estuarine evolution. The magnitude of the ratio a/h may dictate overall tidal asymmetry in shallow estuaries of length common to the U.S. Atlantic coast. For small a/h, virtually all estuaries are ebb-dominant, regardless of extent of tidal flats or marshes. In such systems, frictional drag is insignificantly greater at low water so the delay in estuarine high water due to a relatively small amount of intertidal storage is enough to overcome the influence of friction. For large enough a/h all estuaries will be flood-dominant. In these systems, frictional drag at low tide is too great to be overcome by intertidal storage effects. As suggested by both field data and results of modelling, strongly flood-dominant estuaries are shallow, while strongly ebb-dominant estuaries are relatively deeper. Thus if considered in isolation, the effect of non-linear tidal distortion on estuarine bedload sediment is to make shallow systems shallower and deep systems deeper. Any estuarine system (of the general types discussed) exhibiting pronounced tidal asymmetry can only remain ebb-or flood-dominant, respectively, if future evolution is dependent solely upon the transport of bedload sediment by an unchanging offshore tide and water level.

Boon and Byrne (1981), utilizing a one-dimensional numerical model of tidal inlet and basin systems, suggest that flood-dominant tidal asymmetry can change to ebb-dominant asymmetry with sedimentary infilling of the estuary basin. They argue that in a flooddominant system without tidal flats, high velocity floods may infill an initially deep basin to a level just flooded at high water, increase the area of flats, and eventually produce an ebb-dominant system. Observations in the present study as well as results of the Speer-Aubrey numerical model suggest that for channel lengths characteristic of the U.S. Atlantic coast such an evolution may be possible only for systems with a/h > 0.3 and V_s/V_c < 0.4. Such systems are only weakly flood-dominant with model velocity $M_4/M_2 < 0.1$. An evolution from flood dominance to ebb dominance due to tidal asymmetry would require sedimentary infilling which did not increase a/h, i.e., formation of tidal flats and marshes at the edge of the tidal basin while maintaining consistently deep channels. Whether or not such sedimentation patterns can be formed by weakly flood-dominant tides alone is not clear. For example, estuarine evolution due to tidal asymmetry would require another mechanism at the point where flood dominance switches from flood to ebb dominance and asymmetric distortion is zero. Other forces play a significant role during initial basin infilling, such as supply of suspended and bedload sedments, settling and resuspension of suspended sediment, wind waves and biological factors. Most probably these forces work in conjunction with tidal asymmetry in the initial infilling of an open basin. However, as basin infilling becomes mature in a system where deep channels have been maintained, it is possible that ebb dominance could act increasingly as an inhibitor to further infilling, perhaps to the point of quasi-equilibrium.

In the near future relative sea-level change has the potential for causing dramatic effects upon estuaries throughout the world. If present trends of climatic warming continue, global mean sea level may rise 50 to 170 cm over the next century (NRC, 1979; Hoffman *et al.*, 1983). Field data from existing estuaries and results of numerical modelling suggest that increased water depth reduces the flood-dominant nature of shallow estuaries and, in some cases, may even cause a change to an ebb-dominant system. Thus lower velocity flood and higher velocity ebb currents that would result from accelerated sea-level rise may slow the present rate of natural estuary infilling. Estuaries may expand inland at a faster rate than might be expected otherwise.

Summary and conclusions

Observations of sea-surface elevations and geometric properties of inlet/estuary systems can be utilized to deduce the parameters that govern tidal distortion. Qualitative and semiquantitative effects of variations in these parameters on non-linear tidal sea-surface distortion are reproduced well by the Speer-Aubrey one-dimensional numerical model. Comparison of field observations to model results clarifies the physics of estuarine tidal response and allows the prediction of trends in velocity distortion and near-bed sediment transport. The major findings of this investigation of the parameters that determine tidal distortion in well-mixed estuaries include the following.

(1) In an estuary having little freshwater input, a large tidal amplitude to channel depth ratio (a/h) and long narrow channels, significant overtides and compound tides develop from the dominant offshore equilibrium constituents. The primary interaction in systems examined is with the M_2 tide, whose first harmonic, M_4 , dominates the non-linear signature of the estuary. Growth of M_4 , as measured by the M_4/M_2 surface amplitude ratio, reflects the degree of tidal sea-surface distortion within the estuary.

(2) Depending upon the relative surface phase relationships between M_4 and M_2 , an estuary will have either floods or ebbs of consistently shorter duration. If $0^\circ < 2M_2 - M_4 < 180^\circ$, the estuary is termed 'flood-dominant' and exhibits shorter floods and necessarily higher velocity flood currents. If $180^\circ < 2M_2 - M_4 < 360^\circ$, the estuary is 'ebb-dominant' with shorter, higher velocity ebbs.

(3) Observations of existing estuarine systems along with results of numerical modelling indicate that tidal distortion is a compromise between the effects of frictional distortion in channels and intertidal storage in tidal flats and marshes. Shallow channels slow the propagation of low water through the inner estuary, shortening the flood, whereas extensive intertidal storage slows the propagation of high water, shortening the ebb. (4) Factors other than a/h and V_s/V_c can modify the nature of distortion in tidally dominated shallow estuaries. Surface distortion grows with distance into an estuary, and the longer the length of the entire system the greater the likelihood of flood dominance. Also existing distortion produced on the continental shelf and in semi-enclosed areas such as Sounds may dominate estuarine distortion.

(5) In both actual and model flood-dominant estuaries the degree of sea-surface distortion, as expressed by M_4/M_2 , is controlled primarily by a/h, whereas in ebb-dominant systems it is controlled primarily by V_s/V_c . In each case the first-order relationship is linear, beyond an initial region (a/h ≤ 0.02 and $V_s/V_c \geq 0.2$) of virtually no estuarinederived sea-surface distortion.

(6) For all systems surface $2M_2 - M_4$ decreases with greater a/h. Increased V_s/V_c generally causes $2M_2 - M_4$ to fall in estuaries that are ebb-dominant and rise in those that are flood-dominant. In systems that are very strongly flood- (or ebb-) dominant, surface $2M_2 - M_4$ does not tend toward a perfectly asymmetric 90° (or 270°). Surface $2M_2 - M_4$ continues to fall toward 0° (or 180°) because of slow movement in sea height near low (or high) water with increasing friction (or intertidal storage).

(7) During the spring-neap cycle changes in a/h and V_s/V_c follow approximately linear paths with a/h and V_s/V_c remaining roughly proportional in a single estuary. At virtually all estuaries examined surface M_4/M_2 increases and $2M_2 - M_4$ decreases from spring to neap. Among ebb-dominant systems changes in the distortion parameters are due primarily to increased V_s/V_c , whereas among flood-dominant systems changes are due primarily to greater a/h.

(8) Among strongly flood- (or ebb-) dominant systems, model results indicate surface and velocity relative phase are non-linearly related because velocity $2M_2 - M_4$ does tend toward a perfectly asymmetric 360° (or 180°). For flood-dominant estuaries predicted near-bed sediment transport ratio is primarily an increasing function of a/h, whereas among ebb-dominant estuaries it is an increasing function of V_s/V_c at low a/h and a decreasing function of a/h at moderate a/h.

(9) The magnitude of the a/h ratio alone may dictate overall tidal asymmetry in shallow estuaries of length common to the U.S. Atlantic coast. For small a/h (<0.2), virtually all estuaries are ebb-dominant. For sufficiently large a/h (>0.3) all estuaries are flood-dominant. Only for estuaries where 0.2 < a/h < 0.3 can the system be either moderately flood- or ebb-dominant depending on V_s/V_c

(10) Flood- and ebb-dominant systems engender continued flood- and ebb-dominant behaviour, respectively. Thus strongly flood-dominant systems are unlikely to evolve into strongly ebb-dominant systems by the action of a steady-state offshore tide. Observational and numerical results support the hypothesis of Boon and Byrne (1981) to the extent that infilling of back barrier bays may be slowed to the point of dynamic equilibrium due to strong ebb dominance accompanying increased extent of intertidal flats and marshes. However, if present trends continue, rising global sea level has the potential for giving flood-dominant systems a more ebb-dominant nature and expanding estuaries inland at a faster rate than might otherwise have been expected.

Acknowledgements

This work was supported by the Office of Naval Research through the American Society for Engineering Education, the NOAA National Sea Grant College Program Office, Department of Commerce, under grant NA79AA-D-0102 and NA80-AA-D-00077, WHOI Sea Grant Project No. R/B-46, the Waterways Experiment Station's Coastal Engineering Research Center, the Naval Civil Engineering Laboratory, and the Woods Hole Oceanographic Institution's Coastal Research Center. We thank E. Uchupi and V. A. Fry for their comments.

References

Airy, G. B. 1842 Tides and Waves. Encyclopedia Metropolitan, London. article 192.

Aubrey, D. G. 1986a Hydrodynamic controls on sediment transport in well-mixed bays and estuaries. In Physics of Shallow Estuaries and Bays (van de Kreeke, J., ed.). Springer-Verlag, New York, pp. 245-258.
 Aubrey, D. G. 1986b Kings Bay/Cumberland Sound, Georgia. Part I: experiment report June-July 1985.

Technical Report 86–28, Woods Hole Oceanographic Institution, Woods Hole, MA. 96 pp.

Aubrey, D. G. 1987 Swan Pond River stabilization study. Report submitted to the town of Dennis, MA.

Aubrey, D. G. & Friedrichs, C. T. in press. Seasonal climatology of tidal non-linearities in a shallow estuary. In Hydrodynamics and Sediment Dynamics of Tidal Inlets (Aubrey, D. G. et al., eds). Springer-Verlag, New York.

Aubrey, D. G. & Speer, P. E. 1984 Updrift migration of tidal inlets. Journal of Geology 92, 531-545.

Aubrey, D. G. & Speer, P. E. 1985 A study of non-linear tidal propagation in shallow inlet/estuarine systems. Part I: Observations. *Estuarine, Coastal and Shelf Science* 21, 185–205.

Boon, J. D. III, in press. Temporal variation of shallow-water tides in basin-inlet systems. In Hydrodynamics and Sediment Dynamics of Tidal Inlets (Aubrey, D. G. et al., eds.). Springer-Verlag, New York.

Boon, J. D. III & Byrne, R. J. 1981 On basin hypsometry and the morphodynamic response of coastal inlet systems. *Marine Geology* 40, 27-48.

Boon, J. D. III & Kiley, K. P: 1978 Harmonic analysis and tidal prediction by the method of least squares. Spec. Report No. 186, Virginia Institute of Marine Science, Gloucester Pt., VA. 49 pp.

Boothroyd, J. C. & Hubbard, D. K. 1975 Genesis of bedforms in mesotidal estuaries. In *Estuarine Research*, Vol. 2: *Geology and Engineering* (Cronin, L. E., ed.). Academic Press, New York, pp. 217–234.

Byrne, R. J., Bullock, P. & Tyler, D. G. 1975 Response characteristics of a tidal inlet: a case study. In Estuarine Research, Vol. 2: Geology and Engineering (Cronin, L. E., ed.). Academic Press, New York, pp. 201-216.

DeAlteris, J. T. & Byrne, R. J. 1975 The recent history of Wachapreague Inlet, Virginia. In Estuarine Research, Vol. 2: Geology and Engineering (Cronin, L. E., ed.). Academic Press, New York, pp. 167-181.

Dronkers, J. J. 1964 Tidal Computations in Rivers and Coastal Waters. North Holland Publishing, Amsterdam. 516 pp.

Dronkers, J. J. 1986 Tidal asymmetry and estuarine morphology. Netherlands Journal of Sea Research 20, 117-131.

FitzGerald, D. M. & Nummedal, D. 1983 Response characteristics of an ebb-dominated tidal inlet channel. Journal of Sedimentary Petrology 53, 833-845.

Fry, V. A. 1987 Tidal velocity asymmetries and bedload transport in shallow water embayments. Woods Hole Oceanographic Institution Technical Report 87–51, Woods Hole, MA. 50 pp.

Goud, M. R. & Aubrey, D. G. 1985 Theoretical and observational estimates of nearshore bedload transport rates. *Marine Geology* 64, 91–111.

Hamilton, J. 1978 The quarter-diurnal tide in the English Channel. Geophysical Journal of the Royal Astronomical Society 53, 541-552.

Hoffman, J. S., Keyes D. & Titus, J. G. 1983 Projecting future sea level rise: methodology, estimates to the year 2100, and research needs. *Report 230-09-007*, U.S. Environmental Protection Agency, Washington, DC. 121 pp.

Jarrett, J. T. 1976 Tidal prism-inlet area relationships. G.I.T.I. Report 3, US Army Coastal Engr. Res. Cent., 55 pp.

Masch, F. D., Brandes, R. J. & Reagan, J. D. 1977 Comparison of numerical and physical hydraulic models, Mansonboro Inlet, North Carolina. Appendix 2, v. 1, Numerical Simulation of Hydrodynamics G.I.T.I. Report 6, US Army Coastal Eng. Res. Cent., 123 pp.

Meyer-Peter, E. & Müller, R. 1948 Formulas for bedload transport. In Proceeding of the Second Meeting, International Association of Hydraulic Research. Stockholm, Sweden. pp. 39-64.

NOS (National Ocean Service), 1984 Index of Tide Stations: United States of America and Miscellaneous Other Locations. National Oceanic and Atmospheric Administration, Washington, DC. 140 pp.

NRC (National Research Council), 1979 Carbon Dioxide and Climate: A Scientific Assessment. National Academy Press, Washington, DC. 496 pp.

Nummedal, D. & Humphries, S. M. 1978 Hydraulics and dynamics of the North Inlet, 1975–1976. G.I.T.I. Report 16, US Army Coastal Engr. Res. Cent., 214 pp.

Park, M. 1985 Prediction of Tidal Hydraulics and Sediment Transport Patterns in Stony Brook Harbor. M.S. thesis, SUNY Stony Brook Marine Environmental Science Program, Stony Brook, NY. 146 pp.

tal

ds

nd

In

58.

85.

ry.

ag,

ns. nics ilet es. ch, In rk, ine 81. ng, 20, ıel. ole ort yal the bn, ht., els, Γ.I. ng, her nal Ĩ.I. .S. Parker, B. B. 1984 Frictional Effects on the Tidal Dynamics of a Shallow Estuary. Ph.D. thesis, Johns Hopkins University, Baltimore, MD. 292 pp.

Perry, F. C., Seabergh, W. C. & Lane, E. F. 1978 Improvements for Murrells Inlet, South Carolina. Technical Report H-78-4, US Army Eng. Waterways Exp. Sta., Vicksburg, MI. 339 pp.

Postma, H. 1967 Sediment transport and sedimentation in the marine environment. In *Estuaries* (Lauff, G. H., ed.). American Association for the Advancement of Science Publ. 83, Washington, DC, pp. 158–179.

Salomon, J. C. & Allen, G. 1983 Role sedimentologique de la maree dans les estuaires a fort marnage, Compagnie Francais des Petroles. Notes et Memoires 18, 35-44.

Speer, P. E. 1984 Tidal distortion in shallow estuaries. Ph.D. thesis, WHOI-MIT Joint Program in Oceanography, Woods Hole, MA. 210 pp.

Speer, P. E. & Aubrey, D. G. 1985 A study of non-linear propagation in shallow inlet/estuarine systems. Part II; theory. *Estuarine, Coastal and Shelf Science* 21, 207–224.

Uncles, R. J. 1981 A note on tidal asymmetry in the Severn Estuary. Estuarine, Coastal and Shelf Science 13, 419-432.

Zarillo, G. A. 1985 Tidal dynamics and substrate response in a salt-marsh estuary. Marine Geology 67, 13–35.

Zarillo, G. A. & Park, M. 1987 Sediment transport prediction in a tidal inlet using a numerical model: application to Stony Brook Harbor, Long Island, New York, USA. *Journal of Coastal Research* 3, 429-444.